4-1

**Additional Vocabulary Support**

Using Graphs to Relate Two Quantities

<table>
<thead>
<tr>
<th></th>
<th>axis</th>
<th>decrease</th>
<th>increase</th>
<th>quantities</th>
</tr>
</thead>
</table>

Choose the word from the list above that is defined by each statement.

1. to become greater or larger  
   _____________ increase
2. to become smaller  
   _____________ decrease
3. what each variable is plotted along  
   _____________ axis
4. Amounts that can be determined by measurement, such as degrees.  
   _____________ quantities

Use a word from the list above to complete each sentence.

5. The _____________ axis label tells you what variables are being related.
6. Rising slowly can be described as a(n) _____________ increase.
7. Time and volume are two different _____________ quantities.
8. Falling quickly can be described as a(n) _____________ decrease.
9. When a graph rises, it _____________ increases.
10. You can use graphs to visually represent the relationship between two variable _____________ quantities as they each change.

**Multiple Choice**

11. You climb up a mountain, stop to rest, and then climb back down. Which term best describes the slope of the graph representing your climb back down?  
   B constant  
   △ constant  
   ✘ rising  
   □ increasing

12. You buy two pairs of shoes, and the third pair is free. Which term best describes the slope of the graph showing the cost of the third pair of shoes?  
   F constant  
   △ decreasing  
   ✘ falling  
   □ increasing
Using Graphs to Relate Two Quantities

Skiing  Sketch a graph of each situation. Are the graphs the same? Explain.

a. your speed as you travel from the bottom of a ski slope to the top 
b. your speed as you travel from the top of a ski slope to the bottom

Understanding the Problem

1. What is likely to be true about your speed as you go from the bottom of the ski slope to the top?
   \[ \text{Your speed will be constant since you are on a ski lift.} \]

2. What is likely to be true about your speed as you go from the top of the ski slope to the bottom?
   \[ \text{Your speed will continuously increase if you go straight down.} \]

Planning the Solution

3. What will the graph tend to look like relating to your speed as you go up the ski slope?
   \[ \text{Much of the graph will be a horizontal line.} \]

4. What will the graph tend to look like relating to your speed as you go down the ski slope?
   \[ \text{Much of the graph will be a line with a positive slope.} \]

Getting an Answer

5. Sketch the graph as you travel to the top of the slope.

6. Sketch the graph as you travel to the bottom of the slope.

7. Are the graphs the same? Explain.
   \[ \text{No; going up the ski slope requires riding a ski lift, which maintains a constant speed. If you ski straight down the ski slope, you would keep increasing your speed until you get to the end of the trail.} \]
What are the variables in each graph? Describe how the variables are related at various points on the graph.

1. **Volume of Pool Water**
   - **Time** and **Volume**; The volume increases at a constant rate as time increases.

2. **Temperature of Water**
   - **Depth** and **Temperature**; The temperature decreases at a constant rate as the depth increases.

3. **Plant Height**
   - **Time** and **Plant Height**; The height of a plant increases at a constant rate as time increases.

Match each graph with its related table. Explain your answers.

4. **Distance vs. Time**
   - **Table A**: Time (h) | Distance (mi) | 1 | 60 | 2 | 120 | 3 | 180 | 4 | 240
   - **Graph C**: the graph shows a constant speed of 50 mi/h

5. **Distance vs. Time**
   - **Table B**: Time (h) | Distance (mi) | 1 | 80 | 2 | 125 | 3 | 150 | 4 | 140
   - **Graph B**: the graph shows varying speeds.

6. **Distance vs. Time**
   - **Table C**: Time (h) | Distance (mi) | 1 | 50 | 2 | 100 | 3 | 150 | 4 | 200
   - **Graph A**: the graph shows a constant speed of 60 mi/h
Sketch a graph to represent the situation. Label each section.

7. You buy two shirts. The third one is free.

8. You warm up for gym class, play basketball, and then cool down.

9. The temperature warms up during the day and then decreases at night.

10. **Error Analysis** DVDs cost $19.99 each for the first 2 purchased. After that, they cost $5.99 each. Describe and correct the error in sketching a graph to represent the relationship between the total cost and the number of DVDs purchased.

   The graph indicates that the total cost for 3 DVDs is $5.99, which is not true. The total cost should be $45.97.

11. Sketch a graph of each situation. Are the graphs the same? Explain.
    a. your distance from school as you leave your house and walk to school
    b. your distance from school as you leave school and walk to your house

No; in the first graph, the distance from school is decreasing, and in the second graph it is increasing.
What are the variables in each graph? Describe how the variables are related at various points on the graph.

1. **Graph 1**: Time and temperature; as time increases, the temperature increases.

2. **Graph 2**: Altitude and oxygen level; as the altitude increases, the oxygen level decreases.

Match each graph with its related table.

**Graph 3**: A

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

**Graph 4**: C

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

**Graph 5**: B

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
</tr>
</tbody>
</table>
Sketch a graph to represent the situation.

6. During a trip, your speed increases during the first hour and decreases over the next 2 hours.

7. The average temperature steadily decreases over the course of the football season.

8. The average test score of the class increased throughout the semester until it decreased slightly on the last test.

9. **Error Analysis**  During the first 2 weeks of training, Shelly ran 15 miles per week. Then, she increased to 20 miles per week. Describe and correct the error in sketching a graph to represent the relationship between the weeks and the total number of miles she has run.

   - **The line should continue increasing.**
   - **The third point should be at (3, 50).**
4-1

Standardized Test Prep
Using Graphs to Relate Two Quantities

Multiple Choice

For Exercises 1–3, choose the correct letter.

1. The graph shows your distance from the practice field as you go home after practice. You received a ride from a friend back to his house where you ate supper. You then walked home from there. Which point represents a time when you are walking home? D
   A. A   B. B   C. C   D. D

2. Which table is related to the graph at the right? F
   F.   
   \begin{array}{|c|c|}
   \hline
   \text{Time (h)} & \text{Temp. (°F)} \\
   \hline
   1 & 68 \\
   2 & 73 \\
   3 & 78 \\
   4 & 85 \\
   \hline
   \end{array}
   
   H.   
   \begin{array}{|c|c|}
   \hline
   \text{Time (h)} & \text{Temp. (°F)} \\
   \hline
   68 & 1 \\
   73 & 2 \\
   78 & 3 \\
   85 & 4 \\
   \hline
   \end{array}
   
   G.   
   \begin{array}{|c|c|}
   \hline
   \text{Temp. (°F)} & \text{Time (h)} \\
   \hline
   1 & 85 \\
   2 & 78 \\
   3 & 73 \\
   4 & 68 \\
   \hline
   \end{array}
   
   I.   
   \begin{array}{|c|c|}
   \hline
   \text{Temp. (°F)} & \text{Time (h)} \\
   \hline
   85 & 1 \\
   78 & 2 \\
   73 & 3 \\
   68 & 4 \\
   \hline
   \end{array}

3. How are the variables related on the graph? D
   A. as speed decreases, height stay constant
   B. as speed decreases, height increases
   C. as speed increases, height decreases
   D. as speed increases, height increases

Short Response

4. For the race you swim 1 mile, run 10 miles, and bike 25 miles. Sketch a graph to represent the relationship. Label the axes with the related variables. What are the important points on the graph?
   The important points are the starting and ending points of each activity.
   
   [1] Answer is incomplete.
   [0] Answer is wrong.
Using Graphs to Relate Two Quantities

Scatter plots describe how variables are related. You can also use periodic relationships to show that two variables can be related. Periodic relationships contain patterns that repeat over time. For example, average monthly precipitation varies on a yearly basis. The table shows the average monthly precipitation in Seattle, Washington, and Phoenix, Arizona.

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seattle</td>
<td>5.2</td>
<td>4.1</td>
<td>3.9</td>
<td>2.8</td>
<td>2.0</td>
<td>1.6</td>
<td>0.9</td>
<td>1.2</td>
<td>1.6</td>
<td>3.2</td>
<td>5.7</td>
<td>6.1</td>
</tr>
<tr>
<td>Phoenix</td>
<td>0.8</td>
<td>0.6</td>
<td>0.9</td>
<td>0.3</td>
<td>0.1</td>
<td>0.8</td>
<td>0.8</td>
<td>1.0</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Source: The Weather Channel

1. Draw a scatter plot of the data for Seattle (month, inches) and for Phoenix (month, inches). See points in graph for Exercise 2.

2. Connect each set of data with a curved line.

3. Describe both sets of data. Both sets of data show a repeating pattern. There is more variation in the data for Seattle.

4. Predict what will happen the following year. Extend your graph to show this pattern. The following year will probably yield data similar to the previous year.
Reteaching

Using Graphs to Relate Two Quantities

An important life skill is to be able to read a graph. When looking at a graph, you should check the title, the labels on the axes, and the general shape of the graph.

**Problem**

What information can you determine from the graph?

- The title tells you that the graph describes Trina’s trip.
- The axes tell you that the graph relates the variable of time to the variable of distance to the destination.
- In general, the more time that has elapsed, the closer Trina gets to her destination. In the middle of the trip, the distance does not change, showing she stops for a while.

**Exercises**

What are the variables in each graph? Describe how the variables are related at various points on the graph.

1. **Tiling Job**
   - **Tiles Installed** vs. **Time**
   - **Time and total tiles installed; The number of tiles installed increases as time increases, and then there is a rest during which no tiles are installed, then more tiles are installed, another rest, and then more tiles are installed.**

2. **Dion’s Growth Chart**
   - **Height** vs. **Age**
   - **Age and height; Up until Dion reaches a certain age, his height increases with age at various rates. Then he stops growing.**

3. **Kicked Football**
   - **Height** vs. **Time**
   - **Time and height; When a football is kicked, its height increases with time and then its height decreases with time.**
A graph can show the relationship described in a table.

**Problem**

Which graph shown below represents the information in the table at the right?

Notice that for each additional CD purchased, the total cost increases by $15. The points on the graph should be in a straight line that goes up from left to right. The graph that shows this trend is Graph B.

<table>
<thead>
<tr>
<th>CDs Purchased</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$15</td>
</tr>
<tr>
<td>2</td>
<td>$30</td>
</tr>
<tr>
<td>3</td>
<td>$45</td>
</tr>
<tr>
<td>4</td>
<td>$60</td>
</tr>
<tr>
<td>5</td>
<td>$75</td>
</tr>
</tbody>
</table>

**Exercises**

Match each graph with its related table. Explain your answers.

4. [Graph A] C; as days increase, tickets sold increases
   
5. [Graph B] A; tickets sold decreases until day 4
   
6. [Graph C] B; as days increase, tickets sold decreases

<table>
<thead>
<tr>
<th>Day</th>
<th>Tickets Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day</th>
<th>Tickets Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>65</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day</th>
<th>Tickets Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>2</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
</tr>
</tbody>
</table>
4-2 Additional Vocabulary Support
Patterns and Linear Functions

Concept List

<table>
<thead>
<tr>
<th>dependent variable</th>
<th>function</th>
<th>geometric relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>independent variable</td>
<td>input</td>
<td>linear function</td>
</tr>
<tr>
<td>ordered pairs</td>
<td>output</td>
<td>perimeter</td>
</tr>
</tbody>
</table>

Choose the concept from the list above that best represents the item in each box.

1. (1, 2), (2, 4) - ordered pairs

2. y

3. x

4. geometric relationship

5. Each input is paired with exactly one output value. function

6. independent variable or input

7. output or dependent variable

8. dependent variable or output

9. perimeter

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Electric Car  An automaker produces a car that can travel 40 mi on its charged battery before it begins to use gas. Then the car travels 50 mi for each gallon of gas used. Represent the relationship between the amount of gas used and the distance traveled using a table, an equation, and a graph. Is total distance traveled a function of the gas used? What are the independent and dependent variables? Explain.

Understanding the Problem

1. Describe the miles that the car can travel on the different types of fuel.

   40 mi on a charged battery, 50 mi on one gallon of gas after the battery has been used

Planning the Solution

2. Give a verbal description of the relationship between the miles the car travels and gallons of gas it uses.

   Before using any gas, the car can travel 40 mi. After that, the car travels 50 additional mi/gal.

Getting an Answer

3. Represent this relationship with an equation.

   \[ m = 50g + 40, \text{ where } m = \text{ miles traveled and } g = \text{ gallons of gas} \]

4. Represent this relationship with a table.

<table>
<thead>
<tr>
<th>g</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>190</td>
</tr>
</tbody>
</table>

5. Represent this relationship with a graph.

6. Is total distance traveled a function of the gas used? What are the independent and dependent variables? Explain.

   yes; \( g \) is the independent variable and \( m \) is the dependent variable; the miles traveled depend on the gallons of gas used.
For each diagram, find the relationship between the number of shapes and the perimeter of the figure they form. Represent this relationship using a table, words, an equation, and a graph.

1. 

<table>
<thead>
<tr>
<th>Triangles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>12</td>
<td>( n + 2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Triangles</th>
<th>1 triangle</th>
<th>2 triangles</th>
<th>3 triangles</th>
<th>4 triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>The perimeter is 2 more than the number of triangles; ( p = n + 2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>Squares</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>22</td>
<td>( 2n + 2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Squares</th>
<th>1 square</th>
<th>2 squares</th>
<th>3 squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>The perimeter is 2 more than twice the number of squares; ( p = 2n + 2 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

3. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

yes; the output \( y \) is 1 more than twice the input \( x \); \( y = 2x + 1 \)

4. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

yes; the output \( y \) is 6 more than the input \( x \); \( y = x + 6 \)
For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

5. **Distance Traveled**

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>165</td>
</tr>
</tbody>
</table>

The function is linear; the distance traveled is 55 times the number of hours; \(d = 55t\)

![Graph of Distance Traveled](image)

6. **Calories Burned**

<table>
<thead>
<tr>
<th>Minutes (min)</th>
<th>Calories (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>30</td>
<td>150</td>
</tr>
</tbody>
</table>

The function is linear; the calories burned are 5 times the number of minutes spent exercising; \(c = 5m\)

![Graph of Calories Burned](image)

7. **Reasoning** Graph the set of ordered pairs \((0, 2), (1, 4), (2, 6), (3, 8)\). Determine whether the relationship is a linear function. Explain how you know.

The function is linear; the points on the graph can be connected by a straight line.

8. You can make a bubble solution by mixing 1 cup of liquid soap with 4 cups of water. Represent the relationship between the cups of liquid soap and the cups of bubble solution made using a table, an equation, and a graph. Is the amount of bubble solution made a function of the amount of liquid soap used? Explain. \(b = 5s\);

<table>
<thead>
<tr>
<th>Cups of soap, (s)</th>
<th>Cups of bubble solution, (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>
1. For the diagram below, find the relationship between the number of shapes and the perimeter of the figure they form. Represent this relationship using a table, words, an equation, and a graph.

For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

2. 

<table>
<thead>
<tr>
<th>Rectangles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>22</td>
<td>2n+4</td>
</tr>
</tbody>
</table>

The perimeter of the shapes is twice the number of rectangles plus four. 
\[ P = 2n + 4 \]

For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

2. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Function; each y value is 2 more than the x value; \( y = x + 2 \)

3. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
</tbody>
</table>

Function; each y value is 5 more than twice the x value; \( y = 2x + 5 \)
For each table, identify the dependent and independent variables. Then describe the relationship using words, an equation, and a graph.

4. | $x$ | $y$ |
---|---|---|
0 | -2 |
1 | -1 |
2 | 0 |
3 | 1 |

Independent variable: $x$; dependent variable: $y$; $y = x - 2$; The value of $y$ is the value of $x$ minus 2.

5. | $n$ | $m$ |
---|---|---|
0 | 1 |
1 | -2 |
2 | -5 |
3 | -8 |

Independent variable: $n$; dependent variable: $m$; $m = -3n + 1$; The value of $m$ is $-3$ times the value of $n$ plus 1.

6. **Reasoning**  Graph the set of ordered pairs $(0, 6), (1, 4), (2, 2), (3, 0)$.
Determine whether the relationship is a linear function. Explain how you know.

The relationship is a linear function; the graph of the ordered pairs forms a straight line.
4-2
Standardized Test Prep
Patterns and Linear Functions

Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which equation represents the relationship shown in the table at the right?  
   C. \( y = 2x - 3 \)  
   A. \( y = -x - 3 \)  
   B. \( y = x - 3 \)  
   D. \( y = -2x + 3 \)

2. In a relationship between variables, what is the variable called that changes in response to another variable?  
   I. dependent variable  
   F. function  
   G. input function  
   H. independent variable

3. A lawn care company charges a $10 trip fee plus $0.15 per square foot of \( x \) square feet of lawn for fertilization. Which equation represents the relationship?  
   B. \( y = 0.15x + 10 \)  
   A. \( x = 0.10y + 15 \)  
   C. \( y = 10x + 0.15 \)  
   D. \( x = 10y + 0.15 \)

4. Which equation represents the relationship shown in the graph?  
   I. \( y = \frac{1}{2}x \)  
   G. \( y = 2x \)  
   H. \( y = -\frac{1}{2}x \)  
   F. \( y = -2x \)

Short Response

5. The table below shows the relationship between the number of teachers and the number of students going on a field trip. How can the relationship be described using words, an equation, and a graph?

<table>
<thead>
<tr>
<th>Field Trip</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers</td>
<td>2</td>
</tr>
<tr>
<td>Students</td>
<td>34</td>
</tr>
</tbody>
</table>

The number of students is 17 times the number of teachers; \( s = 17t \);

[1] One or two parts answered correctly.  
[0] No parts answered correctly.
Patterns and Linear Functions

Representing Number Patterns With Linear Equations

Many number patterns can be represented by a linear equation. For example, the positive even integers, 2, 4, 6, 8, . . . , can be represented by the equation $y = 2x$, where $x$ represents the position in the sequence. The fourth even integer ($x = 4$) is $y = 2(4)$ or 8. If you can determine an equation to describe the number pattern, you can use the equation to determine the value of any number in the pattern. For example, the 300th positive even integer is $y = 2(300)$ or 600. Using the equation makes determining the values faster and easier than writing out the entire list.

Exercises

1. Write a linear equation to describe the number pattern 3, 6, 9, . . .  
   $y = 3x$

2. Write a linear equation to describe the number pattern 0, 5, 10, 15, . . .  
   $y = 5x - 5$

3. Write a linear equation to describe the number pattern 4, 7, 10, . . .  
   $y = 3x + 1$

4. Write a linear equation to describe the number pattern that starts with 8 and continues with numbers that are one more than a positive multiple of 7.  
   $y = 7x + 1$

5. Write a linear equation to describe the number pattern that starts with 3 and continues with positive odd integers.  
   $y = 2x + 1$

6. Write a linear equation to describe the number pattern that starts with –2 and continues with negative even integers.  
   $y = -2x$

7. Write a linear equation to describe the number pattern that starts with –8 and continues with negative multiples of 8.  
   $y = -8x$

8. Open-Ended Write a number pattern of your own. Can you write a linear equation to describe the pattern? If so, write the equation. If you cannot determine a linear equation for the pattern, write a new pattern for which you can write a linear equation.
   Answers will vary. Sample answer: 1, 4, 7, 10; yes; $y = 3x - 2$
A relationship can be represented in a table, as ordered pairs, in a graph, in words, or in an equation.

**Problem**

Consider the relationship between the number of squares in the pattern and the perimeter of the figure. How can you represent this relationship in a table, as ordered pairs, in a graph, in words, and in an equation?

Table

For each number of squares determine the perimeter of the figure. Write the values in the table. Remember to focus on the perimeter of the figure, not the squares.

<table>
<thead>
<tr>
<th>Number of squares</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

Ordered Pairs

Let \( x \) represent the number of squares and \( y \) represent the perimeter. Use the numbers in the table to write the ordered pairs.

\[(1, 20), (2, 30), (3, 40), (4, 50), (5, 60)\]

Graph

Use the ordered pairs to draw the graph.

Words

The pattern shows the perimeter is the number of squares times 10 plus 10.

Equation

Write an equation for the words. \[ y = 10x + 10 \]
Exercises

Consider each pattern.

1.

- a. Make a table to show the relationship between the number of trapezoids and the perimeter.

<table>
<thead>
<tr>
<th>Number of trapezoids</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>15</td>
<td>24</td>
<td>33</td>
</tr>
</tbody>
</table>

- b. Write the ordered pairs for the relationship. (1, 15), (2, 24), (3, 33)

- c. Make a graph for the relationship.

- d. Use words to describe the relationship. The perimeter is 6 more than 9 times the number of trapezoids.

- e. Write an equation for the relationship. \( p = 9n + 6 \)

2.

- a. Make a table to show the relationship between the number of cubes and the surface area.

<table>
<thead>
<tr>
<th>Number of cubes</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface area</td>
<td>6</td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

- b. Write the ordered pairs for the relationship. (1, 6), (2, 10), (3, 14)

- c. Make a graph for the relationship.

- d. Use words to describe the relationship. The surface area is 2 more than 4 times the number of cubes.

- e. Write an equation for the relationship. \( s = 4n + 2 \)
Use the list below to complete the Venn diagram.

<table>
<thead>
<tr>
<th>A function whose graph is not a line or part of a line</th>
<th>A function whose graph is a line or part of a line</th>
<th>The graph can be a curve.</th>
</tr>
</thead>
<tbody>
<tr>
<td>These graphs represent constant rates of change.</td>
<td>The points do not lie on a line.</td>
<td>Each input is paired with exactly one output.</td>
</tr>
</tbody>
</table>

Linear Functions
- A function whose graph is a line or part of a line;
- These graphs represent constant rates of change.

Nonlinear Functions
- A function whose graph is not a line or part of a line
- The graph can be a curve.
- The points do not lie on a line.
Fountain A designer wants to make a circular fountain inside a square of grass as shown at the right. What is a rule for the area $A$ of the grass as a function of $r$?

Understanding the Problem

1. What shapes are involved in the areas of grass and the fountain?
   a square and a circle

2. What are the formulas for the areas of these shapes?
   $A = s^2; A = \pi r^2$

Planning the Solution

3. Using $r$ as shown in the drawing, what is a rule for the area of the square?
   $A = 4r^2$

4. Using $r$ as shown in the drawing, what is a rule for the area of the circle?
   $A = \pi r^2$

5. How will you find the area of the remaining grass after the fountain is placed in the grass?
   Subtract the area of the circle from the area of the square.

Getting an Answer

6. What is a rule for the area $A$ of the grass as a function of $r$ after the fountain is placed in the grass?
   $A = 4r^2 - \pi r^2 = (4 - \pi)r^2$
1. A student’s earnings $E$, in dollars, is a function of the number $h$ of hours worked. Graph the function shown by the table. Tell whether the function is \textit{linear} or \textit{nonlinear}.

<table>
<thead>
<tr>
<th>Hours, $h$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings ($), $E$</td>
<td>18</td>
<td>36</td>
<td>54</td>
<td>72</td>
<td>90</td>
</tr>
</tbody>
</table>

\begin{align*}
\text{linear;} \\
\end{align*}

Graph the function shown by each table. Tell whether the function is \textit{linear} or \textit{nonlinear}.

2. \begin{align*}
\begin{array}{c|c}
\hline
x & y \\
\hline
0 & 3 \\
1 & 5 \\
2 & 7 \\
3 & 9 \\
\hline
\end{array}
\end{align*}

\begin{align*}
\text{linear;} \\
\end{align*}

3. \begin{align*}
\begin{array}{c|c}
\hline
x & y \\
\hline
0 & 0 \\
1 & 2 \\
2 & 4 \\
3 & 7 \\
\hline
\end{array}
\end{align*}

\begin{align*}
\text{non-linear;} \\
\end{align*}
Each set of ordered pairs represents a function. Write a rule that represents the function.

4. \((0, 1), (1, 3), (2, 9), (3, 27), (4, 81)\) \(y = 3^x\)

5. \((0, 0), (1, 1), (2, 4), (3, 9), (4, 16)\) \(y = x^2\)

6. \((0, 1), (1, 0.5), (2, 0.25), (3, 0.125), (4, 0.0625)\) \(y = 0.5^x\)

7. \((0, 0), (1, 1), (2, 8), (3, 27), (4, 64)\) \(y = x^3\)

8. **Reasoning** A certain function fits the following description: *As the value of \(x\) increases by 1 each time, the value of \(y\) decreases by the square of \(x\).* Is this function linear or nonlinear? Explain your reasoning.

   nonlinear; There is a squared term in the function.

9. **Writing** The rule \(C = 6.3r\) gives the approximate circumference \(C\) of a circle as a function of its radius \(r\). Identify the independent and dependent variables in this relationship. Explain your reasoning.

   independent: \(r\); dependent: \(C\); The circumference of a circle depends on its radius.

10. **Open-Ended** What is a rule for the function represented by \((0, -2), (1, -1), (2, 2), (3, 7)\)? Explain your reasoning.

   \(y = x^2 - 2\); The graph of the ordered pairs makes it clear that the function is nonlinear. The output is two less than the square of the input.

11. A landscape architect wants to make a triangular garden inside a square of land as shown at the right. What is a rule for the area \(A\) of the garden as a function of \(s\)?

   \(A = \frac{s^2}{2}\)
1. A worker’s wages $W$, in dollars, is a function of the number $h$ of hours worked. Graph the function shown by the table. Tell whether the function is linear or nonlinear.

<table>
<thead>
<tr>
<th>Hours, $h$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages ($), $W$</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

Graph the function shown by each table. Tell whether the function is linear or nonlinear.

2. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

3. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
</tbody>
</table>
Each set of ordered pairs represents a function. Write a rule that represents the function.

4. \((0, 0), (1, 1), (2, 4), (3, 9), (4, 16)\) \(y = x^2\)

5. \((0, 1), (1, 5), (2, 9), (3, 13), (4, 17)\) \(y = 4x + 1\)

6. \((0, -1), (1, 0), (2, 7), (3, 26), (4, 63)\) \(y = x^3 - 1\)

7. \((0, 2), (1, 1), (2, 0), (3, -1), (4, -2)\) \(y = -x + 2\)

8. **Writing** How can you determine if a function is linear or nonlinear from the graph of the function?

   *If the function is linear, the graph will form a straight line. If the function is nonlinear, the graph will not form a straight line.*

9. **Error Analysis** A student says that the function shown by the table below can be represented by the rule \(y = x^2 - 1\). Describe and correct the error.

   \[
   \begin{array}{c|c|c|c|c}
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & -1 & 1 & 3 & 5 & 7 \\
   \end{array}
   \]

   The student used only the first and third sets of points to write the rule; a rule that represents the entire table is \(y = 2x - 1\).
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Which ordered pair represents a linear function?
   A. \((-2, -15), (-1, -9), (0, -3), (1, 3), \) and \((2, 9)\)
   B. \((-2, 4), (-1, 1), (0, 0), (1, 1), \) and \((2, 4)\)
   C. \((-2, -1), (-1, -4), (0, -5), (1, -4), \) and \((2, -1)\)
   D. \((-2, -8), (-1, -1), (0, 0), (1, 1), \) and \((2, 8)\)

2. The following ordered pairs represent a function: \((-2, 10), (-1, 7), (0, 6), (1, 7), \) and \((2, 10)\). Which equation could represent the function?  
   F. \(y = -4x + 2\)
   G. \(y = x^2 - 6\)
   H. \(y = 5x\)
   I. \(y = x^2 + 6\)

3. Which rule could represent the function shown by the table at the right?  
   C. \(y = -x^3\)
   A. \(y = -x^3\)
   B. \(y = x^2 + 1\)
   C. \(y = -x^2 + 1\)
   D. \(y = -x - 1\)

4. The ordered pairs \((-1, 1), (0, 2), (1, 1), (2, -2), \) and \((3, -7)\) represent a function. Which rule could represent the function?  
   G. \(y = -x^2 + 2\)
   F. \(y = -x^2 - 2\)
   H. \(y = x^2 - 2\)
   I. \(y = x^2 + 2\)

5. Which ordered pair represents a nonlinear function?  
   D. \((0, 0), (1, 1), (2, 8), (3, 27), \) and \((4, 64)\)
   A. \((0, 0), (1, 1), (2, 2), (3, 3), \) and \((4, 4)\)
   C. \((0, -1), (1, 0), (2, 1), (3, 2), \) and \((4, 3)\)
   B. \((0, 0), (1, -1), (2, -2), \) and \((4, -4)\)

Short Response

6. Graph the function shown in the table below. Is the function linear or nonlinear?  
   **nonlinear**

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-9</td>
<td>-8</td>
<td>-5</td>
<td>0</td>
</tr>
</tbody>
</table>

   [1] Answer is incomplete.
   [0] Answer is wrong.
4-3 Enrichment
Patterns and Nonlinear Functions

Triangular numbers are numbers that can be represented by a triangular arrangement of dots. The first four triangular numbers are shown.

\[ T_1 = 1, T_2 = 3, T_3 = 6, T_4 = 10 \]

1. Complete the table.

<table>
<thead>
<tr>
<th>Triangular Number</th>
<th>Dots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
</tr>
<tr>
<td>20</td>
<td>210</td>
</tr>
<tr>
<td>100</td>
<td>5050</td>
</tr>
</tbody>
</table>

2. How do you know if a graph of the function represented by the table will be linear or nonlinear?

The graph will be linear if the differences in the number of dots between consecutive triangular numbers is constant.

3. Use the table to make a graph of the first six values.

4. Represent this relationship using an equation.

\[ y = \frac{x^2 + x}{2} \]
If the points of the graph of a function are in a straight line, the function is a **linear function**. If the points of the graph of a function are not in a straight line, the function is a **nonlinear function**.

**Problem**

Is the function given by the table at the right **linear** or **nonlinear**?

Graph the function.

The points are not in a straight line, so the function is nonlinear.

Do you like to solve puzzles? When you are given a list of function values and you are asked to find the rule for the function, you are solving a puzzle. You are looking for a rule that works for all pairs of numbers.

**Problem**

What is a rule that represents the function given by the table below?

Try a rule. Is there an operation or sequence of operations that relates the values in the first column of the table to the values in the second column?

- Try division: $6 \div 2 = 3$, but $8 \div 2 \neq 5$.
- Try another rule. $6 - 3 = 3$ and $8 - 3 = 5$.

Check to make sure this works for all pairs of numbers.

$9 - 3 = 6$ and $12 - 3 = 9$.

The function can be represented by the rule $y = x - 3$. 
4-3  

**Reteaching** (continued)

**Patterns and Nonlinear Functions**

Graph the function shown by each table. Tell whether the function is *linear* or *nonlinear*.

1. $\begin{array}{|c|c|} \hline x & y \\ \hline 0 & 1 \\ 2 & 3 \\ 3 & 4 \\ 6 & 7 \\ \hline \end{array}$  
   linear;  

2. $\begin{array}{|c|c|} \hline x & y \\ \hline 2 & 1 \\ 3 & 3 \\ 4 & 5 \\ 5 & 7 \\ \hline \end{array}$  
   linear;  

3. $\begin{array}{|c|c|} \hline x & y \\ \hline 1 & 4 \\ 2 & 1 \\ 3 & 0 \\ 4 & 1 \\ \hline \end{array}$  
   nonlinear;  

4. $\begin{array}{|c|c|} \hline x & y \\ \hline 2 & 6 \\ 3 & 4 \\ 4 & 3 \\ 6 & 2 \\ \hline \end{array}$  
   nonlinear;  

5. $\begin{array}{|c|c|} \hline x & y \\ \hline -4 & 4 \\ -3 & 3 \\ 0 & 0 \\ 2 & 2 \\ \hline \end{array}$  
   nonlinear;  

6. $\begin{array}{|c|c|} \hline x & y \\ \hline 0 & 1 \\ 2 & 2 \\ 4 & 3 \\ 6 & 4 \\ \hline \end{array}$  
   linear;  

Each set of ordered pairs represents a function. Write a rule that represents the function.

7. $(2, 10), (4, 20), (5, 25), (7, 35), (9, 45)$  
   $y = 5x$  

8. $(2, 5), (4, 9), (5, 11), (7, 15), (10, 21)$  
   $y = 2x + 1$  

9. $(0, 0), (1, 1), (2, 8), (3, 27), (4, 64)$  
   $y = x^3$  

10. $(2, 5), (3, 10), (4, 17), (5, 26), (6, 37)$  
    $y = x^2 + 1$
Complete the vocabulary chart by filling in the missing information.

<table>
<thead>
<tr>
<th>Word or Word Phrase</th>
<th>Definition</th>
<th>Picture or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>continuous function</td>
<td>A continuous function has a graph that is unbroken.</td>
<td></td>
</tr>
<tr>
<td>discrete function</td>
<td>1. A discrete function has a graph composed of isolated points.</td>
<td></td>
</tr>
<tr>
<td>function rule</td>
<td>Describes the relationship between the input and the output.</td>
<td>2. ( y = x - 1 )</td>
</tr>
<tr>
<td>independent variable</td>
<td>3. The value that determines the value of the other variables; the input.</td>
<td></td>
</tr>
<tr>
<td>isolated points</td>
<td>Points that are not connected</td>
<td></td>
</tr>
</tbody>
</table>
**Think About a Plan**

**Graphing a Function Rule**

**Falling Objects**  The height $h$, in feet, of an acorn that falls from a branch 100 ft above the ground depends on the time $t$, in seconds, since it has fallen. This is represented by the rule $h = 100 - 16t^2$. About how much time does it take for the acorn to hit the ground? Use a graph and estimate your answer between two consecutive whole-number values of $t$.

**Understanding the Problem**

1. What do the variables represent in the situation?
   
   $h$ represents the height of the acorn; $t$ represents the number of seconds since the acorn has fallen.

2. What does $h$ equal when the acorn hits the ground?
   
   $h = 0$

**Planning the Solution**

3. How can you determine how much time has elapsed when the acorn hits the ground algebraically?

   **Substitute 0 for $h$ in the equation and solve for $t$.**

4. How will you use a graph to estimate the time?

   **Graph the equation and find the $t$-value where the graph crosses the $h$-axis.**

**Getting an Answer**

5. Graph the function on the grid shown at the right.

6. What two whole-number values is the answer between? What is your estimate? What does this answer mean?

   2 and 3; 2.5; It takes 2.5 s for the acorn to hit the ground.

7. Check your answer algebraically. Show your work.

   
   $0 = 100 - 16t^2$ so $16t^2 = 100$; $t = \sqrt{\frac{100}{16}} = 2.5$ s
Graph each function rule.

1. \( y = 2 - x \)
2. \( y = \frac{1}{2}x \)
3. \( y = 3x + 1 \)

Graph each function rule. Tell whether the graph is **continuous** or **discrete**.

4. The cost \( C \), in dollars, for a health club membership depends on the number \( m \) of whole months you join. This situation is represented by the function rule \( C = 49 + 20m \).

Choose intervals of 1 for the \( m \)-axis because the cost is for every 1 month; Choose intervals of 20 for the \( C \)-axis because that is the cost increment per month; discrete function

5. The cost \( C \), in dollars, for bananas depends on the weight \( w \), in pounds, of the bananas. This situation is represented by the function rule \( C = 0.5w \).

Choose intervals of 1 for the \( w \)-axis because the cost can be measured for every 1 pound; Choose intervals of 0.50 for the \( C \)-axis because that is the cost increment per pound; continuous function
Graph each function rule.

6. \( y = |x| + 1 \)  

7. \( y = x^3 \)

8. \( y = |x| - 2 \)

9. \( y = |x - 1| + 2 \)

10. \( y = -x^2 \)

11. \( y = x^3 - 3 \)

12. **Open-Ended** Sketch a graph of a quadratic function that has \( x \)-intercepts at 0 and 4. **Sample graph:**

13. **Writing** Describe the general shape of the graphs of functions of the form \( y = ax^3 \).

   The function \( y = ax^3 \) passes through the origin with branches in the first and third quadrants. When \( |a| > 1 \), the graph is stretched. When \( 0 < |a| < 1 \), the graph is compressed. When \( a \) is negative, the graph is a reflection in the \( y \)-axis.
Make a table of values for each function. Then graph each function rule.

1. \( y = -x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

Graph each function rule. Explain your choice of intervals on the axes of the graph. Tell whether the graph is continuous or discrete.

4. The cost \( d \), in dollars, for a parking pass depends on the number of whole weeks \( w \) you purchase. This situation is represented by the function rule \( d = 25w \).

The \( w \)-axis interval is 1 because the cost is per 1 week, and the \( d \)-axis interval is 25 because that is the cost increment per week. The function is discrete.

5. The price \( p \), in dollars, for apples depends on the weight \( w \), in pounds, of the apples. This situation is represented by the function rule \( p = 1.99w \).

The \( w \)-axis interval is 1 because the cost is per 1 pound, and the \( p \)-axis interval is 2 because the cost per pound is in increments that are very close to $2 per pound; The function is continuous.
Graph each function rule.

6. \( y = |x| + 3 \)

7. \( y = -3x^2 \)

8. \( y = |x - 2| + 3 \)

9. \( y = -x^2 - 2 \)

10. **Open-Ended** Sketch a graph of a quadratic function. Write the function rule that you graphed.

    *Answers may vary. Sample: \( y = x^2 + 5x \)*

11. **Writing** Describe the general shape of the function \( y = |x| \).
    
    *The general shape of an absolute value function looks like a "V".*
Multiple Choice

For Exercises 1–4, choose the correct letter.

1. Which table of values can be used to graph the function $y = -4x + 3$?  
   
   A. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -1 & -1 \\
   0 & 3 \\
   1 & 7 \\
   2 & 11 \\
   \end{array}
   \]
   
   B. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -3 & -9 \\
   -1 & -1 \\
   1 & 7 \\
   3 & 15 \\
   \end{array}
   \]
   
   C. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 3 \\
   1 & -1 \\
   2 & -5 \\
   3 & -9 \\
   \end{array}
   \]
   
   D. 
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 3 \\
   1 & 7 \\
   2 & 11 \\
   3 & 15 \\
   \end{array}
   \]
   
   C

2. Which term best describes a function whose graph is composed of isolated points?  
   
   F. continuous  
   G. linear  
   H. discrete  
   I. nonlinear
   
   H

3. Which relationship is continuous?  
   
   A. the number of cows a farmer has owned over the years  
   B. the number of cookies Stan baked for the party  
   C. the number of people attending the assembly  
   D. the distance a runner ran during training
   
   D

4. The total cost $c$ a painter charges to paint a house depends on the number $h$ of hours it takes to paint the house. This situation can be represented by the function rule $c = 15h + 245$. What is the total cost if the painter works for 30.25 hours?  
   
   F. $245  
   G. $453.75  
   H. $572.75  
   I. $698.75
   
   I

Short Response

5. The profit $y$ on the number $x$ of items a store sells is represented by the rule $y = 2x - 1$. What does a table of values for the function rule and the graph of the function look like?

   [0] Neither part answered correctly.

The bell curve or “normal curve” is a graph that is shaped like a bell. It lies entirely above the $x$-axis. The area between the curve and the $x$-axis is exactly 1. This area is infinitely wide since the curve never quite touches the axis.

The bell curve at the right shows the average height of men. The far left corner shows the small number of very short men. Moving to the middle of the graph, the curve rises as the majority of the male population is of average height. The right hand corner of the graph shows the small number of particularly tall men. The curve resembles a classic bell shape.

Note that the curve shows only heights from 4’11” to 6’6”. While there are men with heights less than 4’11” and men with heights greater than 6’6”, the number of men at these heights is very small compared to the number of men at heights between 4’11” and 6’6”. The number is so small that it would be difficult to show on this graph.

For Exercises 1–3, use the graph.

1. What is the average height of a man?
   *between 5’8” and 5’9”*

2. Are most men under 5’10”? Explain.
   *yes; more than half of the bell is to the left of 5’10”.

3. Draw a conclusion about how tall most men are.
   *Answers may vary. Sample: Most men are between 5’7” and 5’10”; the area under the curve between 5’7” and 5’10” is greater than half of the total area.*

4. **Reasoning** Describe three real-world relationships that could be represented by a bell curve.
   *Answers may vary. Sample: scores on a test; women’s marathon times; heights of women*
4-4  Reteaching
Graphing a Function Rule

By finding values that satisfy a function rule, you can graph points and discover the shape of its graph.

**Problem**

What is the graph of the function rule \( y = 3x + 5 \)?

First, choose any values for \( x \) and find the corresponding values of \( y \). Make a table of your values.

Then, graph the points from your table. In this case, the points are in a line. Draw the line.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 3x + 5 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( y = 3(-2) + 5 \ = -1 )</td>
<td>(-2, -1)</td>
</tr>
<tr>
<td>-1</td>
<td>( y = 3(-1) + 5 \ = 2 )</td>
<td>(-1, 2)</td>
</tr>
<tr>
<td>0</td>
<td>( y = 3(0) + 5 \ = 5 )</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>1</td>
<td>( y = 3(1) + 5 \ = 8 )</td>
<td>(1, 8)</td>
</tr>
<tr>
<td>2</td>
<td>( y = 3(2) + 5 \ = 11 )</td>
<td>(2, 11)</td>
</tr>
</tbody>
</table>

**Problem**

What is the graph of the function rule \( y = |x - 2| \)?

First, choose any values for \( x \) and find the corresponding values of \( y \). Make a table of your values.

Then, graph the points from your table. In this case, the points make a V shape. Draw the V.

| \( x \) | \( y = |x - 2| \)   | \( (x, y) \)  |
|-------|-------------------|-------------|
| 0     | \( y = |0 - 2| = 2 \) | (0, 2)      |
| 1     | \( y = |1 - 2| = 1 \) | (1, 1)      |
| 2     | \( y = |2 - 2| = 0 \) | (2, 0)      |
| 3     | \( y = |3 - 2| = 1 \) | (3, 1)      |
| 4     | \( y = |4 - 2| = 2 \) | (4, 2)      |
4-4 Reteaching (continued)

Graphing a Function Rule

Exercises

Graph each function rule.

1. \( y = \frac{x}{2} + 3 \)

   \[
   \begin{array}{|c|c|c|}
   \hline
   x & y = \frac{x}{2} + 3 & (x, y) \\
   \hline
   -2 & -1 & (-2, -1) \\
   0 & 3 & (0, 3) \\
   2 & 4 & (2, 4) \\
   \hline
   \end{array}
   \]

2. \( y = -x - 3 \)

   \[
   \begin{array}{|c|c|c|}
   \hline
   x & y = -x - 3 & (x, y) \\
   \hline
   -2 & 1 & (-2, 1) \\
   0 & -3 & (0, -3) \\
   2 & -5 & (2, -5) \\
   \hline
   \end{array}
   \]

3. \( y = x^2 - 3 \)

   \[
   \begin{array}{|c|c|c|}
   \hline
   x & y = x^2 - 3 & (x, y) \\
   \hline
   -2 & 1 & (-2, 1) \\
   0 & -3 & (0, -3) \\
   2 & 1 & (2, 1) \\
   \hline
   \end{array}
   \]

4. \( y = |x| + 1 \)

   \[
   \begin{array}{|c|c|c|}
   \hline
   x & y = |x| + 1 & (x, y) \\
   \hline
   -2 & 3 & (-2, 3) \\
   0 & 1 & (0, 1) \\
   2 & 3 & (2, 3) \\
   \hline
   \end{array}
   \]
There are two sets of note cards below that show how José writes a function rule to find how far a runner can run in 2 hours. The set on the left explains the thinking. The set on the right shows the steps. Write the thinking and the steps in the correct order.

**Think Cards**
- Solve.
- Relate the situation.
- Write the function rule.
- Define your variables.
- Substitute 2 for h.

**Write Cards**
- Let $D = \text{distance}$
- Let $h = \text{number of hours}$
- The runner can run 20 miles in 2 hours.
- $D = 10h$
- Number of miles run is 10 times number of hours.
- $D = 10(2)$

**Think**
- First, relate the situation.
- Second, define your variables.
- Next, write the function rule.
- Then, substitute 2 for $h$.
- Finally, solve.

**Write**
- Step 1 Number of miles run is 10 times number of hours
- Step 2 Let $D = \text{distance}$
- Let $h = \text{number of hours}$
- Step 3 $D = 10h$
- Step 4 $D = 10(2)$
- Step 5 The runner can run 20 miles in 2 hours.
Think About a Plan
Writing a Function Rule

**Projectors** You consult your new projector’s instruction manual before mounting it to the ceiling. The manual says *multiply the desired image width by 1.8 to find the correct distance of the projector lens from the wall.*

a. Write a rule to describe the distance of the lens from the wall as a function of desired image width.

b. The diagram shows the room in which the projector will be installed. Will you be able to project an image 7 ft wide? Explain.

c. What is the maximum image width you can project in the room?

1. What represents the desired image width in the drawing?
   - The “?”

2. What variables will you use in writing the rule and what do they represent?
   - Let \( d \) = the distance between the wall and the lens and let \( w \) = the desired image width.

3. Write a rule to describe the distance of the lens from the wall as a function of desired image width.
   - \( d = 1.8w \)

4. How can you determine if the room is large enough to project an image that is 7 ft wide?
   - **Substitute 7 for \( w \) in the equation and simplify.**

5. Is the room large enough to project an image that is 7 ft wide? Explain.
   - no; \( d = 1.8(7) = 12.6 \)

6. How can you determine the maximum image width that can be projected in this room?
   - **Substitute 12 for \( d \) in the equation and solve for \( w \).**

7. What is the maximum image width you can project in the room? Show your work.
   - \( 6\frac{2}{3} \) ft; \( 12 = 1.8w \) so \( w = 6\frac{2}{3} \)
Write a function rule that represents each sentence.

1. 5 less than one fourth of \( x \) is \( y \).
   \[ \frac{1}{4}x - 5 = y \]

2. 7 more than the quotient of a number \( n \) and 4 is 9.
   \[ \frac{n}{4} + 7 = 9 \]

3. \( P \) is 9 more than half of \( q \).
   \[ P = \frac{1}{2}q + 9 \]

4. 8 more than 5 times a number is \(-27\).
   \[ 5n + 8 = -27 \]

5. 1.5 more than the quotient of \( a \) and 4 is \( b \).
   \[ \frac{a}{4} + 1.5 = b \]

For Exercises 6–10, write a function rule that represents each situation.

6. The price \( p \) of an ice cream is $3.95 plus $0.85 for each topping \( t \) on the ice cream.
   \[ p = 0.85t + 3.95 \]

7. A babysitter’s earnings \( e \) are a function of the number of hours \( n \) worked at a rate of $7.25 per hour.
   \[ e = 7.25n \]

8. The price \( p \) of a club’s membership is $30 for an enrollment fee and $12 per week \( w \) to be a member.
   \[ p = 12w + 30 \]

9. A plumber’s fees \( f \) are $75 for a house call and $60 per hour \( h \) for each hour worked.
   \[ f = 60h + 75 \]

10. A hot dog \( d \) costs $1 more than one-half the cost of a hamburger \( h \).
    \[ d = 0.5h + 1 \]

11. José is 3 years younger than 3 times his brother’s age. Write a rule that represents José’s age \( j \) as a function of his brother’s age \( b \). How old is José if his brother is 5?
    \[ j = 3b - 3; 12 \]

12. A taxicab charges $4.25 for the first mile and $1.50 for each additional mile. Write a rule for describing the total rate \( r \) as a function of the total miles \( m \). What is the taxi rate for 12 miles?
    \[ r = 1.5(m - 1) + 4.25; $20.75 \]
13. Write a function rule for the area of a rectangle whose length is 4 in. more than its width. What is the area of the rectangle when its width is 8 in.?
   \[ A = (w + 4)w; \text{96 in.}^2 \]

14. Write a function rule for the area of a rectangle with a length 3 ft more than two times its width. What is the area of the rectangle when its width is 4 ft?
   \[ A = (2w + 3)w; \text{44 ft}^2 \]

15. Write a function rule for the area of a triangle with a base 2 m less than 4 times its height. What is the area of the triangle when its height is 8 m?
   \[ A = \frac{1}{2} (4h - 2)h; \text{120 m}^2 \]

16. **Reasoning** Write a rule that is an example of a nonlinear function that fits the following description.
   *When b is 49, a is 7, and a is a function of b.*
   
   Answers may vary. Sample: \( a = \sqrt{b} \)

17. **Open-Ended** Describe a real-world situation that represents a nonlinear function.
   
   Answers may vary. Sample: The height of a soccer ball is a function of the time since it was kicked.

18. **Writing** Explain whether or not the relationship between inches and feet represents a function.
   yes; \( y = 12x \), where \( y \) is inches and \( x \) is feet, is a linear function.

19. **Multiple Representations** Use the table shown at the right.
   
   a. Graph the ordered pairs on a coordinate plane.
   
   b. Write an equation that can be used to find \( y \) for any \( x \) value.
      \[ y = 2x + 4 \]
   
   c. Is the equation a function? Explain.
      yes; it is a linear function as the points on the graph can be connected with a straight line.
Write a function rule that represents each sentence.

1. 8 less than one third of x is y. \( y = \frac{1}{3}x - 8 \)

2. 12 more than the quotient of a number \( t \) and 7 is \( v \). \( \frac{t}{7} + 12 = v \)

3. \( z \) is 6 more than twice \( y \). \( z = 2y + 6 \)

4. 10 more than 8 times a number \( a \) is \( b \). \( 8a + 10 = b \)

For Exercises 5–7, write a function rule that represents each situation.

5. The price \( p \) of a large, cheese pizza is $7.95 plus $0.75 for each topping \( t \) on the pizza. \( p = 0.75t + 7.95 \)

6. Jaquelyn’s earnings \( m \) are a function of the number of lawns \( n \) she mows at a rate of $12 per lawn. \( m = 12n \)

7. The total fees \( f \) of a book club membership are $10 per month \( m \) and a one-time administrative fee of $4.75. \( f = 10m + 4.75 \)

8. Eric is 2 years younger than 2 times his sister’s age. Write a rule that represents Eric’s age \( a \) as a function of his sister’s age \( s \). How old is Eric if his sister is 11? \( a = 2s - 2; 20 \)
9. An online music club charges $5.75 for the first music download and $2 for each additional download per month. Write a rule for describing the total monthly fees $f$ as a function of additional downloads $d$. What are the fees for 15 music downloads in a month? 
\[ f = 2d + 5.75; \$33.75 \]

10. Write a function rule for the area of a rectangle whose length is 6 ft more than its width. What is the area of the rectangle when its width is 12 ft?
\[ A = (w + 6)(w) = w^2 + 6w; \ 216 \text{ ft}^2 \]

11. Write a function rule for the area of a rectangle with a length 7 m more than three times its width. What is the area of the rectangle when its width is 3 m?
\[ A = (3w + 7)(w) = 3w^2 + 7w; \ 48 \text{ m}^2 \]

12. Write a function rule for the area of a triangle with a base 10 cm less than 8 times its height. What is the area of the triangle when its height is 5 cm?
\[ A = 4h^2 - 5h; \ 75 \text{ cm}^2 \]

13. **Reasoning** Is the graph of a function that relates a square’s side length to its perimeter continuous or discrete? Explain.
continuous; The function that models this relationship is $P = 4s$, where $P$ is the perimeter of the square and $s$ is the side length. This function is continuous because the side length can be any real number greater than 0.

14. **Open-Ended** Describe a real-world situation that can be represented by a linear function. Describe a change that could occur in this situation that would change it to a nonlinear function.
Answers may vary. Sample: Adding money to a non-interest bearing bank account. If the money goes into a bank account that earns compound interest, then the function would become nonlinear.
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. Jill earns $45 per hour. Using \( p \) for her pay and \( h \) for the hours she works, what function rule represents the situation? B
   A. \( h = 45p \)  B. \( p = 45h \)  C. \( h = p + 45 \)  D. \( p = h + 45 \)

2. What is a function rule for the perimeter \( P \) of a building with a rectangular base if the width \( w \) is two times the length \( l \)? H
   F. \( P = 2l \)  G. \( P = 2w \)  H. \( P = 6l \)  I. \( P = 6w \)

3. Which function rule can be used to represent the area of a triangle with a base \( b \) 8 in. longer than twice the height \( h \) in terms of the height? C
   A. \( A = \frac{1}{2}bh \)  B. \( A = \frac{1}{2}h(h + 8) \)  C. \( A = h^2 + 4h \)  D. \( A = \frac{1}{2}(2h)(h + 8) \)

4. Which equation represents the sentence “\( d \) is 17 less than the quotient of \( n \) and 4”?
   F. \( d = \frac{n}{4} - 17 \)  H. \( d = 4n - 17 \)  G. \( d = \frac{n}{4} + 17 \)  I. \( d - 17 = \frac{n}{4} \)

5. The function rule for the profit a company expects to earn is \( P = 1500m + 2700 \), where \( P \) represents profit and \( m \) represents the number of months the company has been in business. How much profit should the company earn after 12 months in business? D
   A. $15,700  B. $17,700  C. $18,000  D. $20,700

Extended Response

6. A plane was flying at an altitude of 30,000 feet when it began the descent toward the airport. The airplane descends at a rate of 850 feet per minute.
   a. What is the function rule that describes this situation? \( A = 30,000 - 850m \)
   b. What is the altitude of the plane after it has descended for 8 minutes? Show your work. 23,200 ft
   c. Use the function in part a to determine how long it takes for the plane to land if it descends at a continuous rate.

[1] One or two parts answered correctly.
[0] No parts answered correctly.
The inverse of a function is the set of ordered pairs found by swapping the first and second elements of each pair in the original function. If \( f \) is a given function, then \( f^{-1} \) represents the inverse of \( f \). To find an inverse, simply swap the \( x \) and \( y \) coordinates. This new inverse will be a relation, but may not be a function. You can test that inverses are functions by using the vertical line test.

**Problem**

If \( f(x) = x + 2 \), find \( f^{-1}(x) \).

\[
\begin{align*}
  y &= x + 2 & \text{Original equation} \\
  x &= y + 2 & \text{Swap } x \text{ and } y. \\
  y + 2 &= x & \text{Reflexive Property of Equality} \\
  y + 2 - 2 &= x - 2 & \text{Subtract 2 from each side.} \\
  y &= x - 2 & \text{Simplify.}
\end{align*}
\]

So, \( f^{-1} = x - 2 \).

For any ordered pair that makes \( f(x) = x + 2 \) true, the reverse ordered pair will make \( f^{-1} = x - 2 \) true.

### Exercises

1. Graph \( f(x) = x + 2 \) and \( f^{-1} = x - 2 \) to verify they are inverse functions.

Find the inverse of each function. Then graph the function and its inverse.

2. \( f(x) = x - 3 \)  
   \( f^{-1}(x) = x + 3 \)

3. \( f(x) = 2x \)  
   \( f^{-1}(x) = \frac{x}{2} \)
When writing function rules for verbal descriptions, you should look for key words.

<table>
<thead>
<tr>
<th>Words that Suggest Addition</th>
<th>Words that Suggest Subtraction</th>
<th>Words that Suggest Multiplication</th>
<th>Words that Suggest Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>plus</td>
<td>minus</td>
<td>times</td>
<td>divided by</td>
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<tr>
<td>sum</td>
<td>difference</td>
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<td>less than</td>
<td>of</td>
<td>rate</td>
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<td>increased by</td>
<td>decreased by</td>
<td>each</td>
<td>ratio</td>
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<tr>
<td>total</td>
<td>fewer than</td>
<td>factors</td>
<td>half</td>
</tr>
<tr>
<td>in all</td>
<td>subtracted by</td>
<td>twice</td>
<td>a third of</td>
</tr>
</tbody>
</table>

**Problem**

Twice a number \( n \) increased by 4 equals \( m \). What is a function rule that represents the sentence?

\[
\text{twice a number } n \quad \text{increased by } 4 \quad \text{equals} \quad m
\]

\[
2n + 4 = m
\]

The function rule is \( 2n + 4 = m \).

**Exercises**

Write a function rule that represents each sentence.

1. \( t \) is 4 more than the product of 7 and \( s \).
   \[
t = 7s + 4
\]

2. The ratio of \( a \) to 5 equals \( b \).
   \[
a \div 5 = b
\]

3. 8 fewer than \( p \) times 3 equals \( x \).
   \[
3p - 8 = x
\]

4. \( y \) is half of \( x \) plus 10.
   \[
y = \frac{1}{2}x + 10
\]

5. \( k \) equals the sum of \( h \) and 23.
   \[
k = h + 23
\]

6. 15 minus twice \( a \) equals \( b \).
   \[
15 - 2a = b
\]

7. \( m \) equals 5 times \( n \) increased by 6.
   \[
m = 5n + 6
\]

8. 17 decreased by three times \( d \) equals \( c \).
   \[
17 - 3d = c
\]

9. 5 more than the product of 6 and \( n \) is 17.
   \[
6n + 5 = 17
\]

10. \( d \) is 8 less than the quotient of \( b \) and 4.
    \[
d = \frac{b}{4} - 8
\]
4-5  Reteaching (continued)  
Writing a Function Rule

You can write functions to represent situations and then evaluate the function to determine a particular value.

**Problem**

A sales associate earns $500 per week plus 4% of his sales. Write a function rule for the amount he makes in a week if he sells $s$ dollars of merchandise. How much will he make if he sells $4000$ worth of merchandise?

First write the function rule.

\[
\text{earnings} \quad \text{equals} \quad 500 \quad \text{plus} \quad 4\% \text{ of sales}
\]

\[
e = 500 + 0.04s
\]

Use this function rule to calculate how much he will make.

\[
e = 500 + 0.04(4000)
\]

\[
e = 700
\]

He will make $700.

**Exercises**

11. Twelve cans of peaches are placed into each box. Write a function rule for the number of boxes needed for $c$ cans. How many boxes are needed for 1440 cans?

\[
b = \frac{c}{12}; \quad 120 \text{ boxes}
\]

12. Tara plans to rent a car for the weekend. The cost to rent the car is $45 plus $0.15 for each mile she drives. Write a function rule for the total cost of the rental. How much is the rental if she travels 500 miles?

\[
c = 0.15m + 45; \quad 120
\]

13. A plumber charges $60 for a service call plus $55 for each hour she works. Write a function rule for the total bill for a plumbing job. What is the total bill for a job that takes the plumber 3 hours of work?

\[
B = 55h + 60; \quad 225
\]

14. Tickets to a concert cost $45 per ticket plus a $10 processing fee for each order. Write a function rule for the total cost of ordering tickets. What is the total cost to order 6 tickets?

\[
c = 45t + 10; \quad 280
\]
**4-6 Additional Vocabulary Support**

**Concept List**

<table>
<thead>
<tr>
<th>domain</th>
<th>function</th>
<th>function notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>function rule</td>
<td>mapping diagram</td>
<td>not a function</td>
</tr>
<tr>
<td>range</td>
<td>relation</td>
<td>vertical line test</td>
</tr>
</tbody>
</table>

Choose the concept from the list above that best represents the item in each box.

1. \{ (1, 4), (3, 2), (8, 9), (7, 6), (3, 4) \}

   - relation, or not a function

2. \{1, 3, 8, 7\} of \{(1, 4), (3, 2), (8, 9), (7, 6), (3, 4)\}

   - domain

3. Domain \rightarrow Range

   - mapping diagram

4. \( f(x) = -x + 1 \)

   - function notation

5. \[
   x = -1
   \]

   - vertical line test or function

6. \{4, 2, 9, 6\} of \{(1, 4), (3, 2), (8, 9), (7, 6), (3, 4)\}

   - range

7. [Graph of a relation or function]

   - not a function, or relation

8. \[
   \begin{array}{c|c}
   x & y \\
   \hline
   1 & 2 \\
   2 & 3 \\
   3 & 4 \\
   \end{array}
   \]

   - function rule

9. [Graph of a function]

   - function
Car Wash  A theater group is having a carwash fundraiser. The liquid soap costs $34 and is enough to wash 40 cars. Each car is charged $5.

a. If \( c \) is the total number of cars washed and \( p \) is the profit, which is the independent variable and which is the dependent variable?

b. Is the relationship between \( c \) and \( p \) a function? Explain.

c. Write an equation that shows this relationship.

d. Find a reasonable domain and range for the situation.

Understanding the Problem

1. What are the expenses associated with the car wash?
   - the liquid soap, which costs $34

2. If \( c \) is the total number of cars washed and \( p \) is the profit, which is the independent variable and which is the dependent variable? Explain.
   - \( c \) is the independent variable and \( p \) is the dependent variable because the profit depends on the number of cars washed.

Planning the Solution

3. How do you know if a relation is a function?
   - for every input, there is exactly one output

4. How are a reasonable domain and range determined for a function?
   - You must look at the values that make sense for each variable, given what each represents.

5. What limitations does the domain of this function have?
   - The number of cars must be greater than or equal to 0 and less than or equal to 40.

Getting an Answer

6. Is the relationship between \( c \) and \( p \) a function? Explain.
   - yes; there is exactly one output for every input.

7. Write an equation that shows this relationship.
   - \( p = 5c - 34 \)

8. Describe a reasonable domain and range for the situation.
   - The domain is \( 0 \leq c \leq 40 \) and the range is \(-34 \leq p \leq 166\).
Identify the domain and range of each relation. Use a mapping diagram to determine whether the relation is a function.

1. \{ (3, 6), (5, 7), (7, 7), (8, 9) \}  
   \text{Domain: } D = \{ 3, 5, 7, 8 \}; \text{Range: } R = \{ 6, 7, 9 \}  
   \text{The relation is a function.}

2. \{ (0, 0.4), (1, 0.8), (2, 1.2), (3, 1.6) \}  
   \text{Domain: } D = \{ 0, 1, 2, 3 \}; \text{Range: } R = \{ 0.4, 0.8, 1.2, 1.6 \}  
   \text{The relation is a function.}

3. \{ (5, -4), (3, -5), (4, -3), (6, 4) \}  
   \text{Domain: } D = \{ 3, 4, 5, 6 \}; \text{Range: } R = \{ -5, -4, -3, 4 \}  
   \text{The relation is a function.}

4. \{ (0.3, 0.6), (0.4, 0.8), (0.3, 0.7), (0.5, 0.5) \}  
   \text{Domain: } D = \{ 0.3, 0.4, 0.5 \}; \text{Range: } R = \{ 0.5, 0.6, 0.7, 0.8 \}  
   \text{The relation is not a function.}

Use the vertical line test to determine whether the relation is a function.

5.  
   \text{a function}

6.  
   \text{a function}

7. The function \( w(x) = 60x \) represents the number of words you can type in \( x \) minutes. How many words can you type in 9 minutes? 540 words

8. Sound travels about 343 meters per second. The function \( d(t) = 343t \) gives the distance \( d(t) \) in meters that sound travels in \( t \) seconds. How far does sound travel in 8 seconds? 2744 m
Find the range of each function for the given domain.

9. \( f(x) = -3x + 2; \{-2, -1, 0, 1, 2\} \)
   \( R: \{-4, -1, 2, 5, 8\} \)

10. \( f(x) = x^3; \{-1, -0.5, 0, 0.5, 1\} \)
    \( R: \{-1, -0.125, 0, 0.125, 1\} \)

11. \( f(x) = 4x + 1; \{-4, -2, 0, 2, 4\} \)
    \( R: \{-15, -7, 1, 9, 17\} \)

12. \( f(x) = x^2 + 2; \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\} \)
    \( R: \{\frac{2}{16}, \frac{9}{16}, \frac{41}{16}, 3\} \)

Find a reasonable domain and range for each function. Then graph the function.

13. A high school is having a pancake breakfast fundraiser. They have 3 packages of pancake mix that each feed 90 people. The function \( N(p) = 90p \) represents the number of people \( N(p) \) that \( p \) packages of pancake mix feed.

   \[\begin{array}{c|c}
   \text{Packages} & \text{People} \\
   \hline
   0 & 0 \\
   1 & 90 \\
   2 & 180 \\
   3 & 270 \\
   \end{array}\]

   \( D: \) all real numbers \( \geq 0 \) and \( \leq 3; \)
   \( R: \) all real numbers \( \geq 0 \) and \( \leq 270; \)

14. A charter boat travels at a maximum rate of 25 miles per hour. The function \( d(x) = 25x \) represents the distance \( d(x) \), in miles, that the boat can travel in \( x \) hours. The charter boat travels a maximum distance of 75 miles from the shore.

   \[\begin{array}{c|c}
   \text{Distance (mi)} & \text{Hours} \\
   \hline
   0 & 0 \\
   15 & 0.6 \\
   30 & 1.2 \\
   45 & 1.8 \\
   75 & 3.0 \\
   \end{array}\]

   \( D: \) all real numbers \( \geq 0 \) and \( \leq 3; \)
   \( R: \) all real numbers \( \geq 0 \) and \( \leq 75; \)

15. **Reasoning** If \( f(x) = x^2 - 3 \) and \( f(a) = 46 \), what is the value of \( a \)? Explain.

   \(-7 \text{ or } 7; 46 = a^2 - 3, \text{ so } a^2 = 49 \text{ and } a = -7 \text{ or } 7\)

16. **Open-Ended** What is a value of \( x \) that makes the relation \{\((2, 4), (3, 6), (8, x)\)\} a function?

   Answers may vary. Sample: 10
Identify the domain and range of each relation. Use a mapping diagram to determine whether the relation is a function.

1. \{ (2, 4), (8, 11), (9, 1), (4, 2) \}
   - Domain: \{ 2, 4, 8, 9 \}; Range: \{ 1, 2, 4, 11 \}
   - Mapping diagram: yes

2. \{ (5, 2.2), (3, 2.6), (1, 2.6), (0, 2.5) \}
   - Domain: \{ 0, 1, 3, 5 \}; Range: \{ 2.2, 2.5, 2.6 \}
   - Mapping diagram: yes

3. \{ (-4, -6), (1, -2), (-4, 4), (-1, 2) \}
   - Domain: \{ -4, -1, 1 \}; Range: \{ -6, -2, 2, 4 \}
   - Mapping diagram: no

4. \{ (6, 5), (5, 6), (2, 2), (2, 6) \}
   - Domain: \{ 2, 5, 6 \}; Range: \{ 2, 5, 6 \}
   - Mapping diagram: no

Use the vertical line test to determine whether the relation is a function.

5. [Graph showing function]

6. [Graph showing function]
Find the range of each function for the given domain.

7. \( f(x) = -4x + 3; \{ -1, 0, 1, 2, 3 \} \)  
   \{7, 3, -1, -5, -9\}

8. \( f(x) = x^3 + 1; \{ -2, -1, 0, 1, 2 \} \)  
   \{-7, 0, 1, 2, 9\}

9. \( f(x) = x - 6; \{ -5, -3, -1, 1, 3 \} \)  
   \{-11, -9, -7, -5, -3\}

10. \( f(x) = x^2 - 2; \{ -4, -2, 0, 1, 3 \} \)  
    \{14, 2, -2, -1, 7\}

11. A tenth grade class is selling granola bars for a fundraiser. They earn $0.75 for every granola bar that they sell. They have ordered 300 granola bars for the sale. The function \( P(b) = 0.75b \) represents the profit \( P \) the class earns for each bar \( b \) they sell. Find a reasonable domain and range for the function.
    Domain: \( 0 \leq b \leq 300 \); Range: \( 0 \leq P(b) \leq 225 \)

12. The function \( t(x) = 150x \) represents the number of words \( t(x) \) you can speak in \( x \) minutes. How many words can you speak in 20 minutes? 3000

13. Reasoning  If \( f(x) = x^2 - 15 \) and \( f(a) = 49 \), what is the value of \( a \)? Explain.
    \( \pm 8; \) because \( f(\pm 8) = (\pm 8)^2 - 15 = 49 \)

14. Open-Ended  What is a value of \( x \) that makes the relation \( \{(3, 5), (2, 5), (9, x)\} \) a function?
    Answers may vary, but it can be any real number.
4-6 Standardized Test Prep
Formalizing Relations and Functions

Gridded Response
Solve each exercise and enter your answer on the grid provided.

1. What is \( f(-3) \) for the function \( f(x) = -5x - 7 \)? 8

2. You have returned some merchandise to a store and received a store credit of $23. In the same store, you are purchasing picture frames that cost $9 each. The function \( f(x) = 9x - 23 \) represents your total cost \( f(x) \) if you purchase \( x \) picture frames. How many dollars will you pay if you purchase 7 picture frames? 40

3. If \( f(x) = 12x + 14 \), what is the range value for the domain value 3? 50

4. When Jerome travels on the highway, he sets his cruise control at 65 mi/h. The function \( f(x) = 65x \) represents his total distance \( f(x) \) when he has traveled \( x \) hours. How many miles will he have travelled after 3.5 hours of driving on the highway? 227.5

5. For what value of \( x \) is the value of \( f(x) = 4x - 2 \) equal to 18? 5
The exponential function \( y = a \cdot b^x \) can be used to represent exponential growth and exponential decay, where \( x \) represents the time in years. The function represents exponential growth if the value of \( b \) is greater than 1. The function represents exponential decay if the value of \( b \) is less than 1.

For example, if $1000 is invested in an account paying 4% interest each year for two years, the balance in the account at the end of two years is \( y = 1000 \cdot (1.04)^2 \) or $1081.60.

**Exercises**

The population of Smithfield is growing at the rate of 6% per year. There are currently 2500 people in the town.

1. Write an equation that models the population. \( y = 2500(1.06)^x \)

2. Graph the function.

3. Estimate the number of people who will live in Smithfield in 10 years.
   
   **Answers may vary. Sample: about 4500**

The population of Fairbanks is decreasing at a rate of 6% per year. There are currently 2500 people in the town.

4. Write an equation that models the population. \( y = 2500(0.94)^x \)

5. Graph the function.

6. Estimate the number of people who will live in Fairbanks in 10 years.
   
   **Answers may vary. Sample: about 1300**

7. Compare the populations of the towns of Smithfield and Fairbanks. How are they different?
   
   **The town of Smithfield has a population that is increasing exponentially, while the town of Fairbanks has a population that is decreasing exponentially.**
When a relation is represented as a set of ordered pairs, the **domain** of the relation is the set of \( x \)-values. The **range** is the set of \( y \)-values.

A relation where each value in the domain is paired with just one value in the range is called a **function**.

**Problem**

Identify the domain and range of the relation \( \{(−2, 3), (0, 2), (1, 3), (3, 4)\} \).

Represent the relation with a mapping diagram. Is the relation a function?

The domain (or \( x \)-values) is \( \{-2, 0, 1, 3\} \).

The range (or \( y \)-values) is \( \{2, 3, 4\} \).

Notice that each number in the domain is mapped to only one number in the range. This relation is a function.

**Exercises**

Identify the domain and range of each relation. Use a mapping diagram to determine whether the relation is a function.

1. \( \{(2, 3), (4, 6), (1, 5), (2, 5), (0, 5)\} \)
   
   **D:** \( \{0, 1, 2, 4\} \); **R:** \( \{3, 5, 6\} \)

   ![Diagram 1]

   The relation is not a function.

2. \( \{(3, 4), (5, 4), (7, 4), (8, 4), (10, 4)\} \)

   **D:** \( \{3, 5, 7, 8, 10\} \); **R:** \( \{4\} \)

   ![Diagram 2]

   The relation is a function.
You can determine whether or not a relation is a function by looking at the graph of the relation. If a vertical line is drawn anywhere on the graph and passes through two points of the relation, the relation is not a function. This is called the vertical line test.

**Problem**

Is the relation shown below a function? Use a vertical line test.

Notice that two of the dashed vertical lines pass through just one point on the graph.

However, one of the dashed vertical lines passes through three points.

The relation is not a function.

**Exercises**

Use the vertical line test to determine whether the relation is a function.

3. a function  
4. not a function  
5. a function  
6. not a function  
7. not a function  
8. a function
### 4-7 Additional Vocabulary Support

**Arithmetic Sequences**

Use the list below to complete the diagram.

<table>
<thead>
<tr>
<th>Terms</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>arithmetic sequence</td>
<td>each number in a sequence</td>
</tr>
<tr>
<td>sequence formed by adding a fixed number to each previous term</td>
<td>$d$ in $A(n) = A(1) + (n - 1)d$</td>
</tr>
<tr>
<td>each number in a sequence</td>
<td>sequence</td>
</tr>
</tbody>
</table>

- **term of a sequence**
  - each number in a sequence
- **sequence**
  - an ordered list of numbers that often form a pattern
- **common difference**
  - the fixed number in an arithmetic sequence
  - $d$ in $A(n) = A(1) + (n - 1)d$
**Transportation**  Buses on your route run every 9 minutes from 6:00 A.M. to 10:00 A.M. You get to the bus stop at 7:16 A.M. How long will you wait for a bus?

**Understanding the Problem**

1. What is the maximum amount of time you should have to wait for a bus?
   
   9 min

2. How many minutes after the buses begin running at 6:00 A.M. do you arrive at the bus stop?

   76 min

3. What time does the first bus of the day arrive at your bus stop?

   6:00 A.M.

**Planning the Solution**

4. Fill in the table at the right showing the times a bus will stop at your stop.

<table>
<thead>
<tr>
<th>Stop</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6:00</td>
</tr>
<tr>
<td>2</td>
<td>6:09</td>
</tr>
<tr>
<td>3</td>
<td>6:18</td>
</tr>
<tr>
<td>4</td>
<td>6:27</td>
</tr>
<tr>
<td>5</td>
<td>6:36</td>
</tr>
<tr>
<td>6</td>
<td>6:45</td>
</tr>
<tr>
<td>7</td>
<td>6:54</td>
</tr>
<tr>
<td>8</td>
<td>7:03</td>
</tr>
<tr>
<td>9</td>
<td>7:12</td>
</tr>
</tbody>
</table>

5. What is the common difference?

   9

6. According to the table, when will the next bus arrive at your bus stop?

   7:21 A.M.

**Getting an Answer**

7. How long will you wait for a bus?

   5 min
4-7  Practice  Form G

Arithmetic Sequences

Describe the pattern in each sequence. Then find the next two terms of the sequence.

1. 3, 6, 12, 24, . . .
   Each term is twice the previous term; 48, 96

2. 9, 15, 21, 27, . . .
   Each term is six more than the previous term; 33, 39

3. 1.5, 2.25, 3, 3.75, . . .
   Each term is 0.75 more than the previous term; 4.5, 5.25

4. 9.9, 8.8, 7.7, 6.6, . . .
   Each term is 1.1 less than the previous term; 5.5, 4.4

5. 1.5, 4.5, 13.5, 40.5, . . .
   Each term is 3 times the previous term; 121.5, 364.5

6. 40, 20, 10, 5, . . .
   Each term is half the previous term; 2.5, 1.25

7. 7, 11, 15, 19, . . .
   Each term is 4 more than the previous term; 23, 27

8. 67, 60, 53, 46, . . .
   Each term is 7 less than the previous term; 39, 32

9. 12, 7, 2, −3, . . .
   Each term is 5 less than the previous term; −8, −13

Tell whether the sequence is arithmetic. If it is, identify the common difference.

10. 4, 8, 12, 16, . . .
    arithmetic; 4

11. −11, 5, 0, 6, . . .
    not arithmetic

12. 4, 8, 16, 32, . . .
    not arithmetic

13. 12, 23, 34, 45, . . .
    arithmetic; 11

14. 2, 4, 7, 9, . . .
    not arithmetic

15. 1, 3, 9, 27, . . .
    not arithmetic

16. −16, −11, −6, −1, . . .
    arithmetic; 5

17. −9, −4.5, −0.5, 4, . . .
    not arithmetic

18. −7, −14, −21, −28, . . .
    arithmetic; −7

19. 0, $\frac{1}{3}$, $\frac{2}{3}$, 1, . . .
    arithmetic; $\frac{1}{3}$

20. 5, 10, 15, 20, . . .
    arithmetic; 5

    not arithmetic

22. You have a gift card for a coffee shop worth $90. Each day you use the card to get a coffee for $4.10. Write an explicit formula to represent the amount of money left on the card as an arithmetic sequence. What is the value of the card after buying 8 coffees?
   \[ A(n) = 90 − 4.1n; \$57.20 \]

23. You start a savings account with $200 and save $30 each month. Write an explicit formula to represent the amount of money you invest into your savings account as an arithmetic sequence. How much money will you have invested after 12 months?
   \[ A(n) = 200 + 30n; \$560 \]
Find the third, fifth, and tenth terms of the sequence described by each explicit formula.

24. \( A(n) = 4 + (n + 1)(-5) \)  
\(-16, -26, -51\)

25. \( A(n) = 2 + (n + 1)(6) \)  
\(26, 38, 68\)

26. \( A(n) = -5.5 + (n - 1)(2) \)  
\(-1.5, 2.5, 12.5\)

27. \( A(n) = 3 + (n - 1)(1.5) \)  
\(6, 9, 16.5\)

28. \( A(n) = -2 + (n - 1)(5) \)  
\(8, 18, 43\)

29. \( A(n) = 1.4 + (n - 1)(3) \)  
\(7.4, 13.4, 28.4\)

30. \( A(n) = 9 + (n - 1)(8) \)  
\(25, 41, 81\)

31. \( A(n) = 2.5 + (n - 1)(2.5) \)  
\(7.5, 12.5, 25\)

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

32. 1.6, 0.8, 0, –0.8, . . .  
arithmetic; the common difference is \(-0.8\);  
\(A(n) = A(n - 1) - 0.8\);  
\(A(n) = 1.6 + (n - 1)(-0.8)\)

33. 5, 10, 20, 40, . . .  
not arithmetic; there is a common factor, not a common difference

34. 5, 13, 21, 29, . . .  
arithmetic; the common difference is 8;  
\(A(n) = A(n - 1) + 8\);  
\(A(n) = 5 + (n - 1)(8)\)

35. 51, 47, 43, 39, . . .  
arithmetic; the common difference is \(-4\);  
\(A(n) = A(n - 1) - 4\);  
\(A(n) = 51 + (n - 1)(-4)\)

36. 0.2, 0.5, 0.8, 1.1, . . .  
arithmetic; the common difference is 0.3;  
\(A(n) = A(n - 1) + 0.3\);  
\(A(n) = 0.2 + (n - 1)(0.3)\)

37. 7, 14, 28, 56, . . .  
not arithmetic; there is a common factor, not a common difference

38. Open-Ended  Write an explicit formula for the arithmetic sequence whose common difference is \(-2.5\).
Answers may vary. Sample: \( A(n) = 15 + (n - 1)(-2.5) \)

39. Error Analysis  Your friend writes \( A(8) = 3 + (8)(5) \) as an explicit formula for finding the eighth term of the arithmetic sequence 3, 8, 13, 18, . . .
Describe and correct your friend’s error.
The friend is finding the wrong term; \( A(n) = 3 + (n - 1)(5) \) should be the explicit formula, resulting in \( A(8) = 3 + (8 - 1)(5) = 38 \).

40. The local traffic update is given on a radio channel every 12 minutes from 4:00 P.M. to 6:30 P.M. You turn the radio on at 4:16 P.M. How long will you wait for the local traffic update? 8 min

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Describe the pattern in each sequence. Then find the next two terms of the sequence.

1. 15, 11, 7, 3, −1, . . .
   You subtract 4 from each term; −5, −9

2. −2, 2, 6, 10, 14, . . .
   You add 4 to each term; 18, 22

3. 1.5, 3, 4.5, 6, . . .
   You add 1.5 to each term; 7.5, 9

4. 6.8, 5.4, 4, 2.6, . . .
   You subtract 1.4 from each term; 1.2, −0.2

5. 1, 10, 19, 28, . . .
   You add 9 to each term; 37, 46

6. −27, −22, −17, −12, . . .
   You add 5 to each term; −7, −2

Tell whether the sequence is arithmetic. If it is, identify the common difference.

7. 9, 15, 21, 27, . . . yes; 6

8. −14, −10, −7, −3, . . . no

9. −9, 6, 21, 36, . . . yes; 15

10. 1, 5, 7, 9, . . . no

11. 7, −5, −17, −29, . . . yes; −12

12. 72, 48.5, 25, 1.5, . . . yes; −23.5

13. You budget $100 for parking each month. Each day you use the downtown parking lot, it costs you $5. Write an explicit formula to represent the amount of money left in your monthly budget as an arithmetic sequence. How much money is left in your budget after you have used the downtown parking lot 11 times this month?  \[ A(n) = 100 - 5n; A(11) = 45 \]

14. You start an investment account with $3000 and save $100 each month. Write an explicit formula to represent the total amount of money you invest into your account as an arithmetic sequence. How much money will you have invested after 12 months?  \[ A(n) = 3000 + 100n; A(12) = 4200 \]
Find the fourth, sixth, and thirteenth terms of the sequence described by each explicit formula.

15. \( A(n) = 6 + (n - 1)(-2) \)
   \[ A(4) = 0; A(6) = -4; A(13) = -18 \]

16. \( A(n) = 12 + (n - 1)(5) \)
   \[ A(4) = 27; A(6) = 37; A(13) = 72 \]

17. \( A(n) = -2.2 + (n - 1)(-1) \)
   \[ A(4) = -5.2; A(6) = -7.2; A(13) = -14.2 \]

18. \( A(n) = 5 + (n - 1)(0.5) \)
   \[ A(4) = 6.5; A(6) = 7.5; A(13) = 11 \]

19. \( A(n) = -3 + (n - 1)(6) \)
   \[ A(4) = 15; A(6) = 27; A(13) = 69 \]

20. \( A(n) = -7.6 + (n - 1)(3) \)
   \[ A(4) = 1.4; A(6) = 7.4; A(13) = 28.4 \]

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

21. 22, 16, 10, 4, . . .
   Yes, the common difference is -6.
   \[ A(n) = A(n - 1) - 6; \]
   \[ A(n) = 22 + (n - 1)(-6) \]

22. 6, 12, 24, 48, . . .
   No, each term is multiplied by 2.
   \[ A(n) = A(n - 1) + 9; \]
   \[ A(n) = -18 + (n - 1)(9) \]

23. -18, -9, 0, 9, . . .
   Yes, the common difference is 9.
   \[ A(n) = A(n - 1) + 9; \]
   \[ A(n) = -18 + (n - 1)(9) \]

24. 1.5, 2.1, 2.7, 3.3, . . .
   Yes, the common difference is 0.6.
   \[ A(n) = A(n - 1) + 0.6; \]
   \[ A(n) = 1.5 + (n - 1)(0.6) \]

25. Open-Ended  Write an explicit formula for the arithmetic sequence whose common difference is -18.
   Answers may vary. Sample: 0, -18, -36, -54, . . .

26. Reasoning  The initial term of an arithmetic sequence is 5. The eleventh term is 125. What is the common difference of the arithmetic sequence? 12

27. Writing  Explain how you can determine if a sequence is arithmetic.
   If each successive term is found by adding the same number, then the sequence is arithmetic.
Multiple Choice

For Exercises 1–5, choose the correct letter.

1. What are the next two terms of the following sequence? –3, 1, 5, 9, . . . D
   A. –7, –11  B. 10, 11  C. 12, 15  D. 13, 17

2. What are the next two terms of the following sequence? –2, 4, –8, 16, . . . H

3. What is the common difference of the following arithmetic sequence?
   13, –7, –27, –47, . . . A
   A. –20  B. –6  C. –4  D. 20

4. What is the ninth term of the arithmetic sequence defined by the explicit
   formula \( A(n) = -14 + (n - 1)(2) \)? H
   F. –32  G. –30  H. 2  I. 4

5. Each time a touchdown is scored in a football game, 6 points are added to the
   score of the scoring team. A team already has 12 points. What explicit formula
   represents the number of points as an arithmetic sequence? A
   A. \( A(n) = 12 + 6n \)  B. \( A(n) = 12 + (n - 1)(6) \)
   C. \( A(n) = 12 + (n - 1)(6) \)  D. \( A(n) = 12 + (n - 6) \)

Short Response

6. A friend opens a savings account by depositing $1000. He deposits an
   additional $75 into the account each month.
   a. What is an explicit formula that represents the amount of money in the
      account as an arithmetic sequence? \( A(n) = 1000 + 75n \)
   b. How much money is in the account after 18 months? Show your work. $2350

[0] Neither part answered correctly.
A look-and-say sequence begins with a number in which the next term is obtained by describing the previous term. The digits are what you would say as you are writing down the number digit by digit.

121, 111211, 311221, . . .

The first number in the sequence above is 121. This number is read as “one 1, then one 2, then one 1” or “111211.”

Describe 111211. It is read as “three 1’s, then one 2, then two 1’s.”

So, the third digit is three, one, one, two, two, one, or 311221.

1. Find the next three terms of the sequence from above.
   13212211, 111312112221, 31131112213211

2. Find the next three terms of look-and-say sequence 2, 12, 1112.
   3112, 132112, 1113122112

3. Find the next three terms of look-and-say sequence 4, 14, 1114.
   3114, 132114, 1113122114

4. Begin with the number 3. What are the first five terms of this look-and-say sequence?
   3, 13, 1113, 3113, 132113

5. Find the next three terms of look-and-say sequence 53, 1513, 11151113.
   31153113, 132115132113, 11131221151113122113

6. What will be the next term of the look-and-say sequence 22, 22, 22? Explain.
   22; Every term in this sequence is the same.
An orderly list of numbers is called a sequence. Each number in a sequence is called a term. Many sequences follow a pattern. To find the pattern, you will be solving a puzzle.

**Problem**

Describe the pattern of the sequence 8, 4, 0, –4, –8, … . What are the next two terms of the sequence?

You can divide 8 by 2 to get 4, but 4 divided by 2 is not 0. The pattern cannot be “divide by 2.” Look at the pattern again. You can subtract 4 from each number to get the next number.

\[8, 4, 0, -4, -8, \ldots\]

The pattern is “subtract 4 from the previous term.” The next two terms are \(-8 - 4\) or \(-12\) and \(-12 - 4\) or \(-16\).

**Exercises**

Describe the pattern in each sequence. Then find the next two terms of the sequence.

1. 1, 5, 25, 125, …
   multiply the previous term by 5; 625, 3125

2. 3, 9, 15, 21, …
   add 6 to the previous term; 27, 33

3. 64, 32, 16, 8, …
   divide the previous term by 2; 4, 2

4. –5, –3, –1, 1, …
   add 2 to the previous term; 3, 5

5. 1, –3, 9, –27, …
   multiply the previous term by –3; 81, –243

6. 10, 3, –4, –11, …
   subtract 7 from the previous term; –18, –25

7. 1000, –100, 10, –1, …
   divide the previous term by –10; 0.1, –0.01

8. 23, 31, 39, 47, …
   add 8 to the previous term; 55, 63

   multiply the previous term by 3; –324, –972

10. –5, –9, –13, –17, …
    subtract 4 from the previous term; –21, –25

11. 3.6, 4.1, 4.6, 5.1, …
    add 0.5 to the previous term; 5.6, 6.1

12. –81, –27, –9, –3, …
    divide the previous term by 3; –1, \(-\frac{1}{3}\)
An arithmetic sequence is a sequence in which the difference between consecutive terms is constant. This difference is called the common difference. You can find the \( n \)th term of an arithmetic sequence by using the following explicit formula.

\[
A(n) = A(1) + (n - 1)d
\]

In this formula,
- \( A(n) \) represents the \( n \)th term
- \( A(1) \) represents the first term
- \( n \) represents the term number
- \( d \) represents the common difference

**Problem**

Write an explicit formula for the arithmetic sequence 15, 10, 5, 0, \(-5\), \ldots. What is the tenth term of the sequence?

The pattern is to “add \(-5\) to the previous term.”

\[
15, \quad 10, \quad 5, \quad 0, \quad \cdots +(-5) +(-5) +(-5) +(-5)
\]

\[
A(n) = A(1) + (n - 1)d
\]

\[
A(n) = 15 + (n - 1)(-5) \quad A(1) = 15 \text{ and } d = -5
\]

Use the formula to find the tenth term.

\[
A(n) = 15 + (n - 1)(-5)
\]

\[
A(10) = 15 + (10 - 1)(-5) \quad n = 10
\]

\[
A(10) = -30 \quad \text{Simplify.}
\]

**Exercises**

Write an explicit formula for each arithmetic sequence. Then, find the tenth term.

13. 3, 10, 17, 24, \ldots
   \[
   A(n) = 3 + (n - 1)(7); 66
   \]

14. \(-4\), 1, 6, 11, \ldots
   \[
   A(n) = -4 + (n - 1)(5); 41
   \]

15. 44, 40, 36, 32, \ldots
   \[
   A(n) = 44 + (n - 1)(-4); 8
   \]

16. 8, 2, \(-4\), \(-10\), \ldots
   \[
   A(n) = 8 + (n - 1)(-6); -46
   \]

17. 22, 32, 42, 52, \ldots
   \[
   A(n) = 22 + (n - 1)(10); 112
   \]

18. 55, 44, 33, 22, \ldots
   \[
   A(n) = 55 + (n - 1)(-11); -44
   \]
Chapter 4 Quiz 1

Lessons 4-1 through 4-3

Do you know HOW?

Sketch a graph to represent the situation. Label each section.

1. The level of water in a river rose rapidly during the storm and then gradually decreased back to the original level.

2. The volume of a ball increased as more air was added.

For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

3. | x | y |
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<tbody>
<tr>
<td>0</td>
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<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
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</tbody>
</table>

   is a function; the output is 4 times the input, $y = 4x$;

4. | x | y |
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</table>

   not a function;

Each set of ordered pairs represents a function. Write a rule that represents the function.

5. $(0, 5), (1, 6), (2, 7), (3, 8), (4, 9)$ $y = x + 5$
6. $(0, 0), (1, -2), (2, -4), (3, -6), (4, -8)$ $y = -2x$

7. $(0, 2), (1, 5), (2, 8), (3, 11), (4, 14)$ $y = 3x + 2$
8. $(0, 1), (1, 2), (2, 4), (3, 8), (4, 16)$ $y = 2^x$

Do you UNDERSTAND?

9. **Writing** Is the point $(\frac{7}{2}, \frac{3}{2})$ on the graph of $6x - 2y = 18$? How do you know?
   - yes; The ordered pair produces a true statement when the values of $x$ and $y$ are substituted into the equation.

10. **Reasoning** What is the rule for the function represented by $(0, \frac{7}{8}), (1, 2), (2, \frac{25}{8}), (3, \frac{35}{4}), (4, \frac{43}{8})$? Explain your reasoning.
    - $y = \frac{9}{8}x + \frac{7}{8}$; The function is linear with a slope of $\frac{9}{8}$ and a $y$-intercept of $\frac{7}{8}$. 

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Chapter 4 Quiz 2

Do you know HOW?

Write a function rule that represents each sentence.

1. 7 less than three times m is n. \(3m - 7 = n\)
2. 14 more than the quotient of a number t and 10 is u. \(\frac{t}{10} + 14 = u\)
3. 5 times the difference of a number p and 3 is q. \(5(p - 3) = q\)

Identify the domain and range of each relation. Use a mapping diagram to determine whether the relation is a function.

4. \{(3, 1), (5, 7), (8, 9), (10, 12)\}
   - D: \{3, 5, 8, 10\}; R: \{1, 7, 9, 12\}; function
5. \{(3, 2.1), (4, 3.1), (5, 4.1), (6, 5.1)\}
   - D: \{3, 4, 5, 6\}; R: \{2.1, 3.1, 4.1, 5.1\}; function
6. \{(-8, 4), (-10, 5), (-11, 6), (-12, 7)\}
   - D: \{-12, -11, -10, -8\}; R: \{4, 5, 6, 7\}; function
7. \{(4.6, 2.8), (5, 2.2), (5.2, 2), (5.6, 1.8)\}
   - D: \{4.6, 5, 5.2, 5.6\}; R: \{1.8, 2, 2.2, 2.8\}; function

Find the range of each function for the given domain.

8. \(f(x) = 2x + 2; \{-1, 0, 1, 2, 3}\)
   - R: \{0, 2, 4, 6, 8\}
9. \(f(x) = x^2 + 5; \{-3, -1, 0, 2, 4\}\)
   - R: \{5, 6, 9, 14, 21\}
10. \(f(x) = -6x + 5; \{-1, 0, 1, 2, 3\}\)
    - R: \{-13, -7, -1, 5, 11\}
11. \(f(x) = x^2 - 4; \{-2, -1, 0, 3, 4\}\)
    - R: \{-4, -3, 0, 5, 12\}

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

12. 5, 10, 15, 20, … (arithmetic; common diff. is 5; \(A(n) = A(n - 1) + 5\))
13. 200, 100, 50, 25, 12.5, … (not arithmetic; common factor, no common diff. \(A(n) = A(n - 1) + 2.5\))
14. -8.1, -5.8, -3.5, -1.2, … (arithmetic; common diff. is 2.3; \(A(n) = -8.1 + (n - 1)(2.3)\))
15. Write a function rule for the area of a rectangle whose length is 10 ft more than its width. What is the area of the rectangle when its width is 12 ft? \((w + 10)w = w^2 + 10w; 264 \text{ ft}^2\)

Do you UNDERSTAND?

16. Open-Ended Write an explicit formula for an arithmetic sequence that is decreasing. Write a function rule to represent the arithmetic sequence.
   - 20, 18, 16, 14, …; \(A(n) = 20 + (n - 1)(-2)\)
17. Writing Describe how to use the vertical line test to determine whether a graph is a function.
   - If a vertical line passes through more than one point on the graph, then the graph is not a function.
Do you know HOW?

Sketch a graph to represent the situation. Label each section.

1. The temperature of the water decreases over the first few hours in the refrigerator.

2. The sales of the company have increased steadily over the years.

3. The temperature changed as Shelly preheated the oven, cooked the bread, and turned the oven off.

For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

4. \[
\begin{array}{c|c}
 x & y \\
 0 & 4 \\
 1 & 3 \\
 2 & 2 \\
 3 & 1 \\
\end{array}
\]

function; the output is 4 more than the opposite of the input; \( y = -x + 4 \)

5. \[
\begin{array}{c|c}
 x & y \\
 2 & -6 \\
 0 & 0 \\
 -1 & -5 \\
 -1 & 5 \\
\end{array}
\]

not a function

Each set of ordered pairs represents a function. Write a rule that represents the function.

6. \((0, 0), (1, 3), (2, 6), (3, 9), (4, 12)\) \( y = 3x \)

7. \((0, -8), (1, -7), (2, -6), (3, -5), (4, -4)\) \( y = x - 8 \)

8. \((0, -7), (1, -2), (2, 3), (3, 8), (4, 13)\) \( y = 5x - 7 \)

9. \((0, 8), (1, 6), (2, 4), (3, 2), (4, 0)\) \( y = -2x + 8 \)

10. \((-2, \frac{1}{16}), (-1, \frac{1}{4}), (0, 1), (1, 4), (2, 16)\) \( y = 4^x \)

11. \((-2, \frac{10}{9}), (-1, \frac{4}{3}), (0, 2), (1, 4), (2, 10)\) \( y = 3x + 1 \)
Write a function rule that represents each sentence.

12. 1 more than two-thirds of \(a\) is \(b\). \(\frac{2a}{3} + 1 = b\)

13. 11 less than the product of a number \(y\) and \(-2\) is \(z\). \(-2y - 11 = z\)

14. 6 times the sum of a number \(t\) and 5 is \(s\). \(6(t + 5) = s\)

Identify the domain and range of each relation. Use a mapping diagram to determine whether the relation is a function.

15. \(\{(−3, 6), (0, 2), (1, 0), (2, −3)\}\)
   \(D: \{−3, 0, 1, 2\}; \ R: \{−3, 0, 2, 6\}\)
   \(\text{function}\)

16. \(\{(−1, −4), (0, 0), (1, 4), (2, 8)\}\)
   \(D: \{−1, 0, 1, 2\}; \ R: \{−4, 0, 4, 8\}\)
   \(\text{function}\)

Find the range of each function for the given domain.

17. \(f(x) = −2x + 1; \{−2, 0, 2, 4, 6\}\)
   \(R: \{−11, −7, −3, 1, 5\}\)

18. \(f(x) = x^3 + 1; \{−2, −1, 0, 1, 2\}\)
   \(R: \{−7, 0, 1, 2, 9\}\)

19. \(f(x) = −12x − 10; \{−3, −1, 0, 1, 3\}\)
   \(R: \{−46, −22, −10, 2, 26\}\)

20. \(f(x) = x^2 − 7; \{−2, −1, 0, 3, 4\}\)
   \(R: \{−7, −6, −3, 2, 9\}\)

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

21. 15, 11, 7, 3, −1, . . .
   Arithmetic; \(A(n) = A(n − 1) − 4; A(n) = 15 + (n − 1)(−4)\)

22. 7, 10, 14, 19, 25, . . .
   Not arithmetic

23. What is \(f(−5)\) for the function \(f(x) = −9x − 3\)? 42

Do you UNDERSTAND?

24. Reasoning If \(f(x) = −5x + 11\) and \(f(n) = 21\), what is the value of \(n\)?
   Explain.
   
   \(-2; 21 = −5n + 11 \text{ so } 10 = −5n \text{ and } n = −2\)

25. Open-Ended Write the explicit formula for an arithmetic sequence whose tenth term is 75.
   Answers may vary. Sample: \(A(n) = 3 + (n − 1)8\)
Do you know HOW?

Sketch a graph to represent the situation. Label each section.

1. The price of gasoline has steadily increased over the last 10 years.

2. The temperature increased from 9 a.m. to 2 p.m. and then decreased from 2 p.m. to 5:00 p.m.

For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

3. | $x$ | $y$ |
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<td>3</td>
<td>-9</td>
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   Function; the value of $y$ is the value of $x$ multiplied by $-3$; $y = -3x$

4. | $x$ | $y$ |
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   Function; the value of $y$ is the value of $x$ minus 5; $y = x - 5$

Each set of ordered pairs represents a function. Write a rule that represents the function.

5. $(0, 2), (1, 4), (2, 6), (3, 8), (4, 10)$
   $y = 2x + 2$

6. $(0, 1), (1, 3), (2, 9), (3, 27), (4, 81)$
   $y = 3^x$

Do you UNDERSTAND?

7. Open-Ended Write a rule for a nonlinear function such that $y$ is positive for all real number values of $x$. Answers may vary. Sample: $y = x^2 + 1$. 
Chapter 4 Quiz 2

Lessons 4-4 through 4-7

Do you know HOW?

Write a function rule that represents each sentence.

1. 10 more than two times \( x \) is \( y \). \( y = 2x + 10 \)

2. 5 less than the quotient of a number \( n \) and 4 is \( m \). \( \frac{n}{4} - 5 = m \)

Identify the domain and range of each relation.

3. \( \{(5, 4), (7, 6), (9, 8), (11, 10)\} \)  Domain: \( \{5, 7, 9, 11\} \); Range: \( \{4, 6, 8, 10\} \)

4. \( \{(1.1, 2), (2.2, 3), (3.3, 4), (4.4, 5)\} \)  Domain: \( \{1.1, 2.2, 3.3, 4.4\} \); Range: \( \{2, 3, 4, 5\} \)

Find the range of each function for the given domain.

5. \( f(x) = 5x + 1; \{-1, 0, 1, 2, 3\} \) \( \{-4, 1, 6, 11, 16\} \)

6. \( f(x) = x^2 - 3; \{-2, -1, 0, 3, 4\} \) \( \{1, -2, -3, 6, 13\} \)

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

7. 14, 21, 28, 35, \ldots
   Yes, the common difference is 7.
   \( A(n) = A(n - 1) + 7; \)
   \( A(n) = 14 + (n - 1)(7) \)

8. \(-0.5, 0.5, 1.5, 2.5, \ldots\)
   Yes, the common difference is 1.
   \( A(n) = A(n - 1) + 1; \)
   \( A(n) = -0.5 + (n - 1)(1) \)

9. Write a function rule for the area of a rectangle whose length is 5 in. more than its width. What is the area of the rectangle when its width is 8 in.?
   \( A = w^2 + 5w; 104 \text{ in.}^2 \)

Do you UNDERSTAND?

10. Open-Ended  Write an explicit formula for an arithmetic sequence that has 14 as the fourth term.
    Answers may vary. Sample: \( A(n) = 5 + (n - 1)(3) \)

11. Writing  Is the point \((-4, \frac{1}{2})\) on the graph of \( y = \frac{1}{2}x - 4 \)? How do you know?
    No, when you substitute the point into the equation, it is not a solution.
Do you know HOW?

Sketch a graph to represent the situation. Label each section.

1. The average temperature increases throughout the months of April, May, and June.

2. The depth of the water in feet in the pool increased as gallons of water were added.

For each table, determine whether the relationship is a function. Then represent the relationship using words, an equation, and a graph.

3. | x | y |
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<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>-7</td>
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</table>

   Function; the value of \( y \) is the value of \( x \) multiplied by \(-3\) plus 2; \( y = -3x + 2 \)

4. | x | y |
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<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-5</td>
</tr>
</tbody>
</table>

   Function; the value of \( y \) is the value of \( x \) multiplied by 5; \( y = 5x \)

Each set of ordered pairs represents a function. Write a rule that represents the function.

5. \((0, 0), (1, 2), (2, 8), (3, 18), (4, 32)\) \( y = 2x^2 \)

6. \((0, 2), (1, 1), (2, 0), (3, -1), (4, -2)\) \( y = -x + 2 \)
Chapter 4 Test (continued)  

Form K

Write a function rule that represents each sentence.

7. 20 more than one half of \( x \) is \( y \).  \( y = \frac{1}{2}x + 20 \)

8. 9 less than the product of a number \( q \) and 5 is \( r \).  \( 5q - 9 = r \)

Identify the domain and range of each relation.

9. \{(-1, 5), (0, 6), (1, 7), (2, 9)\}
   Domain: \{-1, 0, 1, 2\};  
   Range: \{5, 6, 7, 9\}

10. \{(14, -2), (8, 0), (6, 4), (1, 9)\}
    Domain: \{14, 8, 6, 1\};  
    Range: \{-2, 0, 4, 9\}

Find the range of each function for the given domain.

11. \( f(x) = -x + 5; \{-1, 1, 3, 5, 7\} \)
    \{6, 4, 2, 0, -2\}

12. \( f(x) = x^3 + 2; \{-2, -1, 0, 1, 2\} \)
    \{-6, 1, 2, 3, 10\}

Tell whether each sequence is arithmetic. Justify your answer. If the sequence is arithmetic, write a recursive and an explicit formula to represent it.

13. \(-1, 4, 9, 14, \ldots\)
    Yes, the common difference is 5.  
    \( A(n) = A(n - 1) + 5; \)
    \( A(n) = -1 + (n - 1)(5) \)

14. \(9, 12, 15, 19, \ldots\)
    No, the sequence does not have a common difference.

15. What is \( f(-2) \) for the function \( f(x) = -3x + 8 \)?  \( 14 \)

Do you UNDERSTAND?

16. Reasoning Do the ordered pairs \((-5, 19), (-1, 7), (3, -5), (6, -14), \) and \((9, -23)\) represent a linear function? How do you know?
    Yes, all of the ordered pairs satisfy the equation \( y = -3x + 4 \).

17. Open-Ended Write an example of a sequence that increases in a pattern, but is not an arithmetic sequence. Explain the pattern.
    Answers may vary. Sample: 2, 4, 8, 16, 32, \ldots; You multiply each term by 2 to find the next term.
Performance Tasks

Chapter 4

TASK 1

a. Your friend draws a sequence of dots for an art project as shown. List the number of dots in each of the three figures shown as well as the number of dots that will be in the next three figures. \(1, 4, 9, 16, 25, 36\)
b. State a function rule for the number of dots in each figure. \(A(n) = n^2\)
c. Your friend then draws the sequence of dots shown below. List the number of dots in each of the four figures shown as well as the number of dots that will be in the next two figures. \(1, 3, 6, 10, 15, 21\)
d. State a function rule for the number of dots in each figure. \(A(n) = \frac{n^2 + n}{2}\)

[4] Student shows understanding of the task, completes all portions of the task appropriately with no errors in computation, and fully supports work with appropriate explanations.

[3] Student shows understanding of the task, completes all portions of the task appropriately, with one error in computation, and supports work with appropriate explanations.

[2] Student shows understanding of the task, but makes errors in computation resulting in incorrect answer(s), or needs to explain better.

[1] Student shows minimal understanding of the task or offers little explanation.

[0] Student shows no understanding of the task and offers no explanation.

TASK 2

Invent a story that you can represent with a sketch. (Recall that a graph without actual data is called a sketch.) Check students’ work.

a. Write your story using two variables.

b. Explain why the data is discrete or continuous.

c. Draw a sketch to represent your story.

d. Label each section of the sketch.

[4] Student gives clear and correct diagrams and explanations.

[3] Student gives diagrams and explanations that may contain some minor errors.

[2] Student answers one part correctly and the other part has major errors.

[1] Student gives diagrams or explanations that contain major errors or omissions.

[0] Student makes little or no effort.
Performance Tasks (continued)

Chapter 4

TASK 3

a. Write a paragraph to a friend explaining the meaning of these terms: relation, function, domain, and range.

A relation is a pairing of inputs and outputs. A function is a relation in which there is exactly one output for every input. The set of inputs is the domain and the set of outputs is the range.

b. Draw graphs of several different functions. Then draw two graphs of relations that are not functions. Label each example. Check students’ work.

c. Choose one of the function graphs you drew above. Create a table of values. Write a function rule for the graph. Check students’ work.

d. Evaluate the function rule \( y = 3x^2 + 4 \) for \( x = -2 \). Explain which is the dependent variable and which is the independent variable. Graph the function. \( y = 3(-2)^2 + 4 = 16; x \) is the independent variable and \( y \) is the dependent variable.

<table>
<thead>
<tr>
<th>Student performance</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Student shows understanding of the task, completes all portions of the task appropriately, and fully supports work with appropriate graphs explanations.</td>
</tr>
<tr>
<td>3</td>
<td>Student shows understanding of the task, completes all portions of the task appropriately, and supports work with appropriate graphs and explanations with a minor error.</td>
</tr>
<tr>
<td>2</td>
<td>Student shows understanding of the task, but needs to explain better.</td>
</tr>
<tr>
<td>1</td>
<td>Student shows minimal understanding of the task or offers little explanation.</td>
</tr>
<tr>
<td>0</td>
<td>Student shows no understanding of the task and offers no explanation.</td>
</tr>
</tbody>
</table>

TASK 4

a. Write the first five terms of an arithmetic sequence that has a positive common difference. Answers may vary. Sample: 4, 7, 10, 13, 16

b. Write an explicit formula rule that describes the sequence from part (a).

Answers may vary. Sample: \( A(n) = 3(n - 1) + 4 \)

c. Write the first five terms of an arithmetic sequence that has a negative common difference. Answers may vary. Sample: 2, -5, -12, -19, -26

d. Write an explicit formula rule that describes the sequence from part (c).

Answers may vary. Sample: \( A(n) = -7(n - 1) + 2 \)

e. Write the first five terms of a sequence that is not arithmetic. Answers may vary. Sample: 1, 2, 4, 7, 11

[3] Student gives explanations that may contain some minor errors.
[2] Student answers one part correctly and the other part has major errors.
[1] Student gives explanations that contain major errors or omissions.
[0] Student makes little or no effort.
Cumulative Review
Chapters 1–4

Multiple Choice
For Exercises 1–8, choose the correct letter.

1. Which statement is equivalent to $4x - 6y + 8y - 6x - 12y$? C
   A. $10x - 26y$  B. $12x - 18y$  C. $-2x - 10y$  D. $-12y$

2. Jacob scored 5 points more than $\frac{4}{5}$ of Sarai’s test score. Which equation represents the relationship between Jacob’s test score, $J$, and Sarai’s test score, $S$? G
   F. $\frac{4}{5} J + 5 = S$  G. $\frac{4}{5} S + 5 = J$

3. The height of the flagpole, $F$, and the length of its shadow, $S$, are proportional to Harry’s height, $h$, and the length of his shadow, $l$. Which proportion represents the relationship? A
   A. $\frac{F}{h} = \frac{S}{l}$  B. $\frac{S}{F} = \frac{l}{h}$

4. Which ordered pair is a solution of $y = -6x - 7$? G
   F. $(0.5, -4)$  G. $(2, -19)$  H. $(-2, -5)$  I. $(3, 11)$

5. What is the value of $15 - (6^2 - 5^2)$? D
   A. $-104$  B. $-74$  C. $74$  D. $4$

6. Which values of $x$ and $y$ will make the expression $3(-2x - y)^2$ equal to 3? H
   F. $x = -2$ and $y = 1$  G. $x = -2$ and $y = -3$
   H. $x = -1$ and $y = 3$  I. $x = -1$ and $y = -3$

7. The sum of two consecutive integers is 153. Which equation can be used to find the first integer $n$? A
   A. $2n + 1 = 153$  B. $2n = 153$  C. $n + 1 = 153$  D. $2n + 2 = 153$

8. Which property is illustrated by $a(b + c) = ab + ac$? I
   F. Associative Property of Addition  G. Associative Property of Multiplication
   H. Commutative Property of Addition  I. Distributive Property
Cumulative Review (continued)

Chapters 1–4

9. Simplify $3(-5x - 7) - (6x)$.
   $-21x - 21$

10. Admission to the movie is $7.50$ for adults and $3.50$ for students. The theater’s
goal is to receive $1500$ in revenue for the evening. Write an equation that
models this relationship.
   $7.5a + 3.5s = 1500$

11. Evaluate $-3a(2b - 4c)$ for $a = -3$, $b = 2$, and $c = -7$.
    288

12. The profit of a company is modeled by $P = 4t + 15,500$, where $P$ is the profit
and $t$ is the time in months. What is the dependent variable? $P$

13. Use words to describe the relationship modeled by the equation
   $y = -8x + 17$.
   $y$ is equal to $17$ more than $-8$ times $x$.

14. Simplify $9^3 + 3^3 - 5(8 - 4) + 9^2$.
    88

15. Vocabulary What is the term for a relation in which each input value
    corresponds to exactly one output value?
    function

16. Simplify $(ab^2 + 10 + a) - (6ab^2 - 2ab + 8)$.
    $-5ab^2 + 2ab + a + 2$

17. Write a recursive and an explicit formula to represent the following arithmetic
    sequence: $2, -5, -12, -19, \ldots$
    $A(n) = A(n - 1) - 7; A(n) = 2 + (n - 1)(-7)$

18. What is the value of the expression $(-8)(6) - (-4)(-5) + (-3)(-6)$?
    $-50$

19. A student wrote a number pattern such that each number is $3$ more than $4$
times the previous number. What is the sixth term in the pattern if the first
term is $6$?
    7167

20. One lap around a track measures $400$ meters. Kelvin runs $8$ laps around the
    track for a race. How many kilometers did he run?
    3.2 km
Chapter 4 Project Teacher Notes: Fast Talker

About the Project

Using tongue twisters provides students with a fun way to gather data and to use functions and graphs. The activities in this project will give students practice at graphs and tables as they investigate and display relationships among sets of data.

Introducing the Project

- Have students work with partners or in small groups. Select one of these tongue twisters or choose one of your own: The sunshade sheltered Sarah from the sunshine or Lavonne lingered, looking longingly for her lost laptop. Tell students how many words there are in the tongue twister you have chosen. Then have each group predict how long it will take you to say it.
- As you read it, have the groups record the time. Ask groups to compare their predicted times with the actual recorded time. Choose another tongue twister and repeat. Instruct students to create a table and graph to record the data they are gathering.
- Encourage the students to research more tongue twisters, investigate why some are “easier” to say than others, and study the effect of practice on speed.

Activity 1: Doing

Students choose a tongue twister and then time each other saying it. They combine other data with their own to create a table and graph.

Activity 2: Graphing

Have students time another tongue twister, create a table, and construct a graph.

Activity 3: Writing

Students evaluate the given function and create a table and graph.

Activity 4: Analyzing

Students select a new tongue twister and collect new data. Then they construct a graph to determine if it is linear and write a function rule to predict the time to say the tongue twister.

Finishing the Project

You may wish to plan a project day on which students share their completed projects. Encourage groups to explain their processes as well as their results. Have students review their project work and update their folders.
- Have students review the equations, graphs, and explanations that they needed for the project.
- Ask groups to share their insights that resulted from completing the project, such as any shortcuts they found for writing equations or making graphs.
Chapter 4 Project: Fast Talker

Beginning the Chapter Project

Radio announcers must time their speeches so that commercials and news updates are the correct length. Do you know how fast you talk? How fast do your friends talk? Try saying these tongue twisters: The sunshade sheltered Sarah from the sunshine and Lavonne lingered, looking longingly for her lost laptop.

As you work through the activities, you will time people as they say tongue twisters. You will use graphs to help investigate and display relationships in the data you collect. Then, using functions, you will summarize your findings and make predictions.

List of Materials
- Calculator
- Stopwatch
- Graph paper

Activities

Activity 1: Doing

Work in a group. Choose a tongue twister from the top of the page.
- Time one person saying the tongue twister. Record the time to the nearest tenth of a second.
- Time two people saying the tongue twister. Be sure they speak one after the other.
- Repeat with more people. Make a table for the data you collect.
- Collect data from other groups.
- Plot the points for all the data. Does there seem to be any relationship between the number of people and the time? Explain.

Activity 2: Graphing

Sopchoppy, Florida, is the home of the Sopchoppy Shoe Shop.
- Time the tongue twister The Sopchoppy Shoe Shop sells shoes with a group of at least ten other people. Use the same method you used in Activity 1.
- Record your data in a table and display the data in an appropriate graph.
Chapter 4 Project: Fast Talker (continued)

Activity 3: Writing

The function \( t(n) = 4.3n \) predicts the time \( t \) (in seconds) it takes \( n \) people in a row to say the tongue twister, *A cricket critic cricked his neck at a critical cricket match*.

- Find \( t(5) \). Explain what it represents.
- What does \( t(0) = 0 \) mean?
- Suppose \( t(n) = 34.4 \) seconds. How can you determine how many people in a row said the tongue twister?
- Make a table of values for the function and graph it.

Activity 4: Analyzing

Think of a tongue twister not yet used in the activities. You may know one in a language other than English.

- With a group of at least ten people, use the same method to record times as you used in Activity 1.
- Graph the data. Do your data points form a line? Explain.
- Write a function rule that you could use to predict the time for \( n \) people to say your tongue twister.

Finishing the Project

The answers to the four activities should help you complete your project. Present your project as a poster or other visual display. For each tongue twister, show either a table and a graph, or a function rule with a graph. Your presentation should discuss input and output values and how a function rule can be used to predict the time it will take \( n \) people to consecutively repeat a tongue twister.

Reflect and Revise

Show your project to an adult and review your work together. Be sure your tables, graphs, and other work are easy to follow. Check that you have included all the required information. Consider adding artwork or illustrations to your project. Make any revisions necessary to improve your work.

Extending the Project

Most people cannot sustain a rate of speech greater than 300 words/min. Experiment with your own rate of speech. Consider ways in which you could measure it. You might do so by comparing rates for a familiar memorized speech, reading aloud, and saying tongue twisters. Determine how you could express these rates as functions.
Chapter 4 Project Manager: Fast Talker

Getting Started

Read the project. As you work on the project, you will need a stopwatch, calculator, graph paper, materials on which you will record your calculations, and materials to make accurate and attractive graphs. Keep all of your work for the project in a folder.

Checklist

☐ Activity 1: timing tongue twisters
☐ Activity 2: recording and graphing data
☐ Activity 3: calculating function values
☐ Activity 4: writing a function rule
☐ visual display

Suggestions

☐ Organize the data as you collect it.
☐ You need a group of at least ten people and should choose the type of graph suited to the data.
☐ Show the steps of your work.
☐ Look for a pattern in your data.
☐ Does there appear to be a clear relationship between the data? Would you be confident making predictions based on your function rule? Why or why not?

Scoring Rubric

3  All work is complete. Accurate graphs and charts are made and labeled correctly. Computations are correct. Explanations are clear. A function rule corresponds to the data.
2  Graphs and tables are mostly accurate and have useful labels. Most calculations are correct, with only minor errors. Explanations and the function rule make sense.
1  Graphs and tables need additional information. Computations and written explanations are partially correct.
0  Major elements of the project are incomplete or missing.

Your Evaluation of Project  Evaluate your work, based on the Scoring Rubric.

Teacher’s Evaluation of Project